Journal of Computational Mathematics, Vol.21, No.1, 2003, 15–24.

EXACT NONREFLECTING BOUNDARY CONDITIONS FOR AN ACOUSTIC PROBLEM IN THREE DIMENSIONS*1)

Houde Han Chunxiong Zheng
(*Department of Mathematical Sciences, Tsinghua University, Beijing 100084, China*)

Dedicated to the 80th birthday of Professor Zhou Yulin

Abstract

In this paper, nonreflecting artificial boundary conditions are considered for an acoustic problem in three dimensions. With the technique of Fourier decomposition under the orthogonal basis of spherical harmonics, three kinds of equivalent exact artificial boundary conditions are obtained on a spherical artificial boundary. A numerical test is presented to show the performance of the method.

Key words: Exterior problem, Nonreflecting artificial boundary condition, Acoustic equation, Three-dimensions.

1. Introduction

Numerical simulation has been of immense importance in the research fields of acoustics, such as sound propagation and sound scattering. Mathematically, these problems usually lead to some PDE defined on an unbounded domain. When one tries to devise a numerical scheme, a great difficulty occurs owing to the unboundedness of the physical domain. Since most of the numerical methods require a finite computational domain, a natural idea is to introduce some artificial boundary to limit the unbounded domain, and then set up some kind of boundary condition on the artificial boundary. This is just the basic idea of the so-called *artificial boundary method*. Generally, this boundary condition should be chosen carefully so that the problem restricted to the bounded domain is not only well-posed, but is also a highly accurate approximation of the original problem.

In the last several decades, mathematicians have made splendid progress in this method. Consider the example of wave-like equations. Here, Engquist and Majda [3] derived the artificial boundary conditions with the Padé approximation of the pseudodifferential operator on a line-type artificial boundary for the hyperbolic equation. Bayliss and Turkel [2] obtained a series of artificial boundary conditions based on asymptotic expansion of the solution for the same hyperbolic equation at large distance. Higdon [6] considered the two-dimensional hyperbolic equation in a rectangular computational domain. He designed a series of artificial boundary conditions which are perfectly absorbing for plane waves in some prescribed wave directions. All these artificial boundary conditions are local in both time and space.

Well-designed nonlocal artificial boundary conditions have the potential of being more accurate than any local one. Grote and Keller [5] once designed an exact artificial boundary condition on a spherical artificial boundary for a hyperbolic equation in three dimensions (the linear scalar acoustic equation belongs to this category). By decomposing the function into the sum of spherical harmonics, they obtained an exact boundary condition for each harmonic

* Received September 30, 2002.
1)This work is supported partly by the Special Funds for Major State Basic Research Projects of China and the National Science Foundation of China.

component by a suitable change of the unknowns. Then they combine all the components to get the final answer. Based on the Laplace transform, one can easily obtain the exact artificial boundary condition on a circular artificial boundary for the exterior problem of the scalar hyperbolic equation. But an integral kernel is involved in this boundary condition, which is the inverse Laplace transform of the logarithmic derivatives of a Hankel function. How to deal with this kernel becomes the key problem in its numerical implementation. This is just the problem considered in the paper of Alpert, Greengard and Hagstrom [1]. They use the technique of pole expansion to approximate the kernel by a series of exponential functions for any prescribed accuracy.

In this paper, we concentrate on the design of exact nonreflecting artificial boundary conditions for an acoustic problem. We propose a new method which is much different from the one of Grote and Keller [5].

2. Setup of the considered problem

Consider the setup shown in Figure 1. The shaded region with boundary Γ denotes some sound source, such as a vibrating drum. The generated wave propagates in the exterior domain $\hat{\Omega}$ outside the source boundary Γ. If the data on Γ has been detected, we want to solve the pressure field. In the real application, only a certain finite region close to the source, say Ω, is of "physical interest". We assume that Ω is bounded by Γ and a spherical surface B of radius R, which, in most of the literature, is called the artificial boundary. Finally , we define $D = \hat{\Omega} \backslash \bar{\Omega}$ to be the residual unbounded domain.

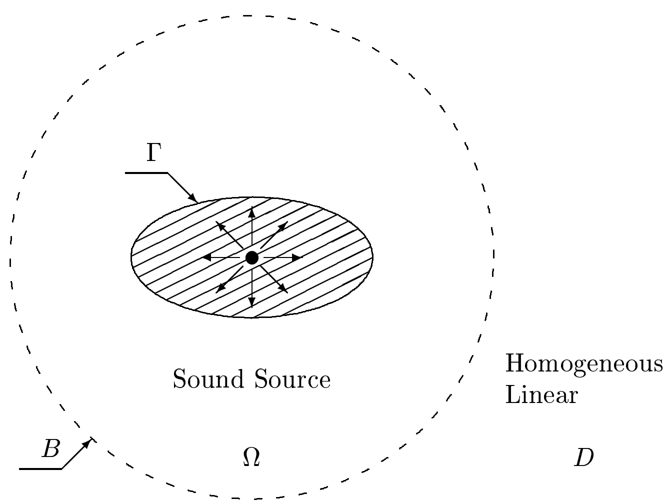

Figure 1: Setup of the considered sound propagation problem.

In order to apply the proposed method in the following, some regularity assumptions should be made on the unbounded domain D and the pressure field. In particular, we assume that the medium in D is homogeneous and behaves linearly; the pressure field is of sufficient smoothness and has zero initial value and null source on a open domain containing the closure of D. On the other hand, no such assumptions are necessary in Ω. The media can be even inhomogeneous and nonlinear. The reason for different assumptions, which will be clear at the end of the paper, is that we have to solve a problem analytically in D to build up a proper relation between the unknown and some of their derivatives on B, whereas the problem in Ω is only intended to be solved numerically after this relation is imposed on B. Any limitation to the generality of this