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THE MORTAR ELEMENT METHOD FOR ROTATED $Q1$ ELEMENT*1)

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Abstract

In this paper, a mortar element version for rotated $Q1$ element is proposed. The optimal error estimate is proven for the rotated $Q1$ mortar element method.

Key words: Mortar element method, Rotated $Q1$ element.

1. Introduction

Many authors have made significant contributions to the so-called mortar element method (see [4] [5] [7] [8] [10] [11], and references therein). The mortar element method is a nonconforming domain decomposition method with non-overlapping subdomains. The meshes on different subdomains need not align across subdomain interfaces, and the matching of discretizations on adjacent subdomains is only enforced weakly. This offers the advantages of freely choosing highly varying mesh sizes on different subdomains and is very promising to approximate the problems with abruptly changing diffision coefficients or local anisotropies.

The rotated $Q1$ element is an important nonconforming element. It was first proposed and analysed in [12] for numerically solving the Stokes problem. The rotated $Q1$ element provides the simplest example of discretely divergence-free nonconforming element on quadrilaterals. Due to its simplicity, the rotated $Q1$ element is used to simulate the deformation of martensitic crystals with microstructure in [9]. Independently, it also was derived within the framework of mixed element method (see [2]). In [2] it was proven that Raviart-Thomas mixed rectangle element method is equivalent to rotated $Q1$ nonconforming element method.

The purpose of this paper is to study the rotated $Q1$ mortar element method. A mortar element version for rotated Q1 element is proposed. By constructing some relations between rotated $Q1$ mortar element and bilinear element, the optimal error estimate for rotated $Q1$ mortar element method is proven.

The remainder of this paper is organized as follows. In §2 we introduce model problem, the rotated $Q1$ mortar element method, and some notations. In §3 some technical Lemmas are given. In §4 the optimal error estimate is shown. For convenience, the symbols $\preceq$, $\succeq$, and $\asymp$ will be used in this paper, and $x_1 \preceq y_1$, $x_2 \succeq y_2$, and $x_3 \asymp y_3$ mean that $x_1 \le C_1 y_1$, $x_2 \ge c_2 y_2$, and $c_3 x_3 \le y_3 \le C_3 x_3$ for some constants C_1, c_2, c_3, and C_3 that are independent of mesh

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parameters. For any subdomain $D \subset \Omega$, we use usual L^2 inner product $(\cdot,\cdot)_D$, Sobolev space $H^s(D)$ with usual Sobolev norm $\|\cdot\|_{H^s(D)}$ and seminorm $|\cdot|_{H^s(D)}$. If $D=\Omega$, we denote the usual L^2 inner product by $(\cdot,\cdot)$, the Sobolev norm by $\|\cdot\|_s$ and seminorm by $|\cdot|_s$, where s may be fractional (for details see [1]).

2. Preliminaries

Consider the following model problem: find $u \in H_0^1(\Omega)$ such that

$$a(u,v) = f(v), \quad \forall v \in H_0^1(\Omega), \tag{2.1}$$

where

$$a(u,v) = (\bigtriangledown u, \bigtriangledown v), \quad f(v) = (f,v),$$

$f \in L^2(\Omega)$, Ω is a rectangular or L-shape bounded domain.

Divide Ω into geometrically conforming rectangular substructures, i.e., $\bar{\Omega} = \bigcup\limits_{k=1}^{N} \bar{\Omega}_k$ with $\bar{\Omega}_k \cap \bar{\Omega}_l$ being empty set or a vertex or an edge for $k \neq l$. With each Ω_k we associate a quasi-uniform triangulation $\mathcal{T}_h(\Omega_k)$ made of elements that are rectangles whose edges are parallel to x-axis or y-axis. The mesh parameter h_k is the diameter of the largest element in $\mathcal{T}_h(\Omega_k)$. Let Γ_{kl} denote the open edge that is common to Ω_k and Ω_l. Denote by Γ the set of all interfaces between the subdomains, i.e., $\Gamma = \bigcup \partial\Omega_k \backslash \partial\Omega$. Each edge inherits two triangulations made of segments that are edges of elements of the triangulations of Ω_k and Ω_l respectively. In this way each Γ_{kl} is provided with two independent and different one dimensional meshes, which are denoted by $\mathcal{T}_h^k(\Gamma_{kl})$ and $\mathcal{T}_h^l(\Gamma_{kl})$ respectively. Let $\Omega_{k,h}$ and $\partial\Omega_{k,h}$ be the sets of vertices of the triangulation $\mathcal{T}_h(\Omega_k)$ that are in $\overline{\Omega}_k$ and $\partial\Omega_k$ respectively.

For each triangulation $\mathcal{T}_h(\Omega_k)$, the rotated $Q1$ element space is defined by

$$\begin{aligned} X_h(\Omega_k) = \{v \in L^2(\Omega_k) \quad | \quad & v|_E = a_E^1 + a_E^2 x + a_E^3 y + a_E^4(x^2 - y^2), \\ & a_E^i \in \mathcal{R}, \ \int_{\partial E \cap \partial\Omega} v|_{\partial\Omega} ds = 0, \ \forall E \in \mathcal{T}_h(\Omega_k); \\ & \text{for } \ E_1, E_2 \in \mathcal{T}_h(\Omega_k), \ \text{ if } \ \partial E_1 \cap \partial E_2 = e, \ \text{ then} \\ & \int_e v|_{\partial E_1} ds = \int_e v|_{\partial E_2} ds\}, \end{aligned}$$

with norm and seminorm

$$\|v\|_{H_h^1(\Omega_k)} = (\sum_{E \in \mathcal{T}_h(\Omega_k)} \|v\|^2_{H^1(E)})^{1/2}, \quad |v|_{H_h^1(\Omega_k)} = (\sum_{E \in \mathcal{T}_h(\Omega_k)} |v|^2_{H^1(E)})^{1/2}.$$

Introduce the global discrete space

$$X_h(\Omega) = \prod_{k=1}^{N} X_h(\Omega_k),$$

with norm $\|v\|_{1,h} = (\sum\limits_{k=1}^{N} \|v\|^2_{H_h^1(\Omega_k)})^{1/2}$ and seminorm $|v|_{1,h} = (\sum\limits_{k=1}^{N} |v|^2_{H_h^1(\Omega_k)})^{1/2}$.