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ASYMPTOTIC STABILITY FOR GAUSS METHODS FOR NEUTRAL DELAY DIFFERENTIAL EQUATIONS*1)

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Abstract

In [4] we proved that all Gauss methods are$N\tau(0)$-compatible for neutral delay differential equations (NDDEs) of the form :

$$\begin{aligned} &y'(t) = ay(t) + by(t-\tau) + cy'(t-\tau), \quad t > 0, \\ &y(t) = g(t), \qquad -\tau \le t \le 0, \end{aligned} \tag{0.1}$$

where a, b, c are real, $\tau > 0$, $g(t)$ is a continuous real valued function. In this paper we are going to use the theory of order stars to characterize the asymptotic stability properties of Gauss methods for NDDEs. And then proved that all Gauss methods are $N\tau(0)$-stable.

Key words: Delay differential equations, Stability, Runge-Kutta methods.

1. Introduction

In the past, most of the work on the asymptotic stability for delay and neutral delay differential equations dealt with finding the stability region independently of the delay term. Al Mutib[1] and recently N. Guglielmi [8, 9, 10] revisited the investigation of stability region for a fixed but arbitrary delay term for so called $\tau(0)$-stability. Some results have been pointed out for DDEs, which have been reeaximined for NDDEs [4]. It has already been shown [7] all Gauss methods are $\tau(0)$-stability for DDEs. In this paper we pursue our investigation of Gauss methods in a NDDEs case. In order t simplify the notation, without loosing the generality of the problem we can fix the delay equal to 1. For the sake of the simplicity, in the sequel we deal with the following test equation

$$\begin{aligned} &y'(t) = ay(t) + by(t-1) + cy'(t-1), \quad t > 0, \\ &y(t) = g(t), \qquad -1 \le t \le 0, \end{aligned} \tag{1.1}$$

where a, b, c are real, $\tau > 0$, $g(t)$ is a continuous real valued function. Its characteristic equation is given by:

$$\lambda - a - b\exp(-\lambda) - c\lambda\exp(-\lambda) = 0. \tag{1.2}$$

It is known that the set of triplet $(a,\ b,\ c)$ for which the solution $y(t)$ of (1.1) tend to zero when $t \to \quad \infty$ is given by:

$$\Sigma_* = \{(a,b,c) \in I\!R^3 \text{ all root } \ \lambda \ \text{ of (1.2) satisfying Re } [\lambda] < 0, |c| < 1\}.$$

It can be rewriten as $\Sigma_* = \Sigma \cup \mathrm{E}$ where

$$\mathrm{E} = \{(a,b,c) \in I\!R^3, \quad a + |b| < 0 \quad \text{and} \quad |c| < 1\,\},$$

$$\Sigma = \left\{(a,\ b, c) \in I\!R^3,\ |a| < -b, \text{ and } \sqrt{b^2 - a^2} < \sqrt{1 - c^2}\arccos\left(\frac{c + \rho}{1 + c\rho}\right) \text{ with } |c| < 1\right\}.$$

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with $\rho = -\frac{a}{b}$. This set is bounded in the right by the plane
$P = \{(a, b, c) \in I\!R^3,\ a = -b \text{ with } a < 1 - c, |c| < 1\}$ and the transcendental surface

$$\Gamma_* = \{(a_*(\theta, c), b_*(\theta, c), c) \in I\!R^3 \mid \theta \in (0, \pi) \text{ and } a < 1 - c, |c| < 1\},$$

with

$$a_*(\theta, c) = \frac{\theta \cos\theta - c\theta}{\sin\theta}, \qquad b_*(\theta, c) = \frac{c\theta \cos\theta - \theta}{\sin\theta}.$$

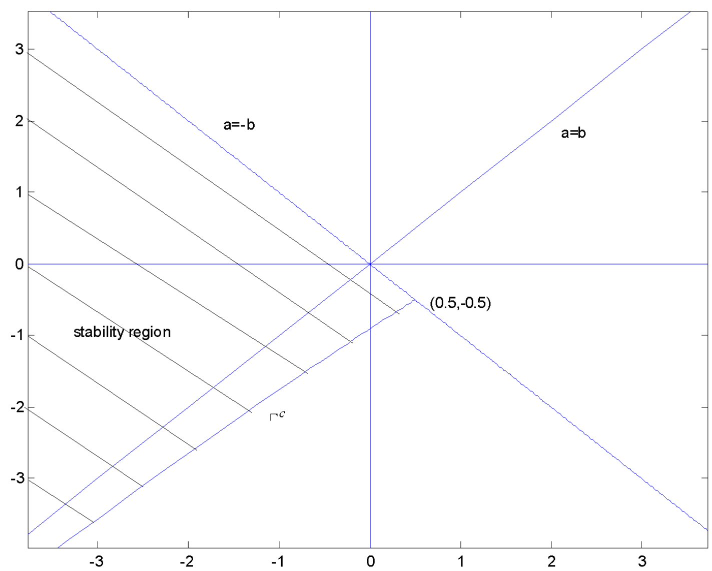

Figure 1. Stability region of analytical solution of equation (1.1) for $c = 0.5$

1.1. Runge -Kutta methods for NDDEs

Let us consider the following s-stage RK method

$$y_{n+1} = y_n + h \sum_{i=1}^{v} w_i K_i^{n+1} \tag{1.3}$$

$$K_i^{n+1} = f\left(t_n + c_i h,\ y_n + h \sum_{j=1}^{v} a_{ij} K_j^{n+1},\ y_{n-m} + h \sum_{j=1}^{v} b_{ij} K_j^{n-m+1},\ \sum_{j=1}^{v} c_{ij} K_j^{n-m+1}\right),$$

$i = 1, 2, \ldots, s$, where $h = \tau/m$, $c_i = \sum_{j=1}^{s} a_{ij}$. Here $W = [w_1, \ldots, w_s]^T$ and the matrix $A = [a_{ij}]_{i,j=1}^{s}$ define a RK method for ODEs. [3, 6, 13]. The second argument in f can be interpreted as an approximation to $y(t)$ at the intermediate point $t_n + c_j h$. Similarly the third argument in f can be interpreted as an approximation to $y(t_{n-m} + c_j h)$ and the fourth to $y'(t_{n-m} + c_j h)$ usually $b_{ij} = w_j(c_i)$ and $c_{ij} = w_j'(c_i)$ where $w_i(\theta)$, $i = 1, \ldots s$ are polynomials which define the natural continuous extension of RK method, i.e. polynomials such that the approximate