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BIVARIATE LAGRANGE-TYPE VECTOR VALUED RATIONAL INTERPOLANTS*[1)]

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Abstract

An axiomatic definition to bivariate vector valued rational interpolation on distinct plane interpolation points is at first presented in this paper. A two-variable vector valued rational interpolation formula is explicitly constructed in the following form: the determinantal formulas for denominator scalar polynomials and for numerator vector polynomials, which possess Lagrange-type basic function expressions. A practical criterion of existence and uniqueness for interpolation is obtained. In contrast to the underlying method, the method of bivariate Thiele-type vector valued rational interpolation is reviewed.

Key words: Bivariate vector value, Rational interpolation, Determinantal formula.

1. Introduction

Wynn [11] proposed a method for rational interpolation of vector-valued quantities given on a set of distinct interpolation points. He used continued fractions and generalized inverses for the reciprocal of vector-valued quantities. McCleod [9] pointed out that Wynn's proof of the termination of a continued fraction representation of a rational function requires that the underlying field be algebraically closed. He provided a solution to the dilemma by noting that the algebraic operations used in Wynn's proof are valid if N is restricted to be any associative division algebra over the complex filed $A(C)$. Using Thiele fractions interpreted with generalized inverses as follows:

$$\vec{v}^{-1} = 1/\vec{v} = \vec{v}^*/|\vec{v}|^2, \quad \vec{v} \neq 0, \vec{v} \in C^d \tag{1.1}$$

Graves-Morris [7] proved that an interpolating fraction

$$\vec{R}(x) = \vec{b}_0 + \frac{x - x_0}{\vec{b}_1} + \cdots + \frac{x - x_{n-1}}{\vec{b}_n}$$

may normally be found for vector data $\{(x_i, \vec{v}_i) : i = 0, 1, \cdots, n\}$, where $\vec{v} \in C^d, \vec{b}_i \in C^d, x_i \in R$.

Gu Chuanqing [3-4] , Zhu Gongqin and Gu Chuanqing [5] showed that by means of the convergents of Thiele -type branched continued fractions for two-variable functions [10], the generalized inverse (1.1) may be used to define bivariate Thiele-type vector valued rational interpolants (see(5.1) and (5.2)) for vector data

$$\{\vec{v}_{i,j} : \vec{v}_{i,j} = \vec{v}(x_i, y_j) \in C^d, (x_i, y_j) \in \widetilde{Z}_{n,m}\} \tag{1.2}$$

where $\widetilde{Z}_{n,m} = \{(x_i, y_j) : i = 0, 1, \cdots, n, j = 0, 1, \cdots, m, x_i, y_j \in R\}$ be a rectangular grid contained in R^2 and each finite vector $\vec{v}_{i,j}$ is associate with a distinct plane interpolation point (x_i, y_j).

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Graves-Morris and Jenkins [8] presented an axiomatic approach to vector valued rational interpolation in the one-variate case. They constructed interpolants for vector-valued data so that the components of the resulting vector valued rational interpolant share a common denominator polynomial. An explicit determinantal formula for denominator polynomials was given for the denominator polynomial for vector valued rational interpolation on distinct real or complex points. In this paper, an axiomatic definition to bivariate vector-valued rational interpolation on distinct plane interpolation points is at first presented. A two-variable vector valued rational interpolation formula is explicitly constructed in the following form: the determinantal formulas for denominator scalar polynomials and for numerator vector polynomials, which possess Lagrange-type basic function expressions. A practical criterion of existence and uniqueness for interpolation is obtained. Some examples are given to illustrate the results in this paper. In the end, in contrast to the underlying method, the method of bivariate Thiele-type vector valued rational interpolation([4],[5]) is reviewed.

2. Definition

Two-variable, generalized inverse vector valued rational interpolants discussed by this paper obey some basic principles, which was at first put forward by Graves-Morris [7] in the one-variate case, as follows:

(i) If, for some fixed $k, k = 1, 2, \cdots, d$, the kth components of the vectors $\vec{v}_{i,j}$ is the only non-zero components, then the vector valued interpolant reduces to the corresponding rational fraction interpolant .

(ii) The value of the vector rational interpolant does not depend on the order in which the interpolation points are used to construct the interpolant.

(iii) There is some sense in which a specified rational interpolant is unique.

(iv) The poles of the d components of the vector interpolant normally occur at common positions in the xoy-plane.

Given the date set as (1.2), Let the interpolation set $\widetilde{Z}_{n,m}$ change to

$$Z_{n,m} = \{(x_i, y_j) : i = 0, 1, \cdots, n, j = 0, 1, \cdots, m, x_i \in C, y_j \in C\}$$

in (1.2).

Definition 2.1. *For vector data (1.2) with $Z_{n,m}$, the generalized inverse vector valued rational interpolant ($BGIRI_L$) of type [n+m/n+m] is a vector of rational function*

$$\vec{R}(x, y) = \vec{P}(x, y)/q(x, y), \tag{2.1}$$

where $\vec{P}(x, y) = (p^{(1)}(x, y), \cdots, p^{(d)}(x, y)) \in C^d$ is a complex vector polynomial, $q(x, y)$ is a scalar polynomial, satisfying the following conditions:

(i) $\partial\{\vec{P}\} = \max_{1\le k\le d} \partial\{p^{(k)}\} \le n + m, \quad \partial\{q\} = n + m,$ (2.2)

(ii) $q(x, y)|\vec{P}(x, y) \cdot \vec{P}^*(x, y),$ (2.3)

(iii) $q(x, y) = q^*(x, y),$ (2.4)

(iv) $\vec{R}(x_i, y_j) = \vec{v}_{i,j}, \quad (x_i, y_j) \in Z_{n,m}, q(x_i, y_j) \ne 0,$ (2.5)

where a superscript $*$ denotes complex conjugate and the dot product between elements used in (2.3) is usually defined by $\vec{u} \cdot \vec{v} = u^{(1)}v^{(1)} + \cdots + u^{(d)}v^{(d)}$.

3. Construction

The $ijth$ cardinal polynomial of Lagrange-type is defined by

$$l_{ij}(x, y) = \prod_{u=0,u\ne i}^{n} \frac{x - x_u}{x_i - x_u} \prod_{v=0,v\ne j}^{m} \frac{y - y_v}{y_j - y_v}, \quad (x_i, y_j) \in Z_{n,m}. \tag{3.1}$$