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DOMAIN DECOMPOSITION METHODS WITH NONMATCHING GRIDS FOR THE UNILATERAL PROBLEM*1)

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Abstract

This paper is devoted to the construction of domain decomposition methods with nonmatching grids based on mixed finite element methods for the unilateral problem. The existence and uniqueness of solution are discussed and optimal error bounds are obtained. Furthermore, global superconvergence estimates are given.

Key words: Domain decomposition, Variational inequality, Global superconvergence, Nonmatching grids.

1. Introduction

Domain decomposition methods (DDMs) with nonmatching grids, which have been developed in recent years, are a quite new class of nonconforming DDMs. As this kind of DDMs can be applied to solving many practical problems which can't be handled by using the usual DDMs, they are been earning particular attention of computational experts and engineers. The advantege of these methods is that they allow non-coincidence of the nodal points at common edges or common faces. Thus they can deal with the problems of moving grids and can design the optimal meshes, namely, one can choose different mesh-sizes and different orders of approximate polynomials in different subdomains according to the different properties of solutions and different requirements of practical problems.

The superconvergence estimates for the finite element methods have been developed in the last ten years. Its mathematical framework is being perfected. We refer to Křižek and Neittanmäki [14], Lin and Xu [16], Lin and Zhu [17,24], Křižek [15] and Wahlbin [23] for details.

The finite element approximations and error analysis for the unilateral problem were studied in many papers. We refer to Brezzi, Hager and Raviart [4,5], Glowinski, Lions and Trémoliérés [9], Haslinger [10], Haslinger and Hlavácek [11,12], Kikuchi and Oden [13] for more details.

We will discuss in this paper the domain decomposition methods with nonmatching grids for the unilateral problem. The nonconforming on the interface of subdomains is handled by introducing Lagrange multipliers. The finite element analysis of the mixed formulation for the unilateral problem is presented and error estimates are derived. Furthermore, we give the global superconvergences if the partition of domain Ω is almost a uniform piecewise strongle regular mesh and the solution is smooth enough.

Let Ω be a bounded domain in R^n with Lipschitz boundary. We will use the usual Sobolev space $W^{m,p}(\Omega)$ consisting of real valued functions defined on Ω with derivatives through order m in $L^p(\Omega)$ and the norm on $W^{m,p}(\Omega)$ is denoted by $||\cdot||_{m,p,\Omega}$. In particular, we define

$$H^m(\Omega) = W^{m,2}(\Omega) \quad \text{and} \quad ||\cdot||_{m,\Omega} = ||\cdot||_{m,2,\Omega}.$$

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Let us consider the following unilateral problem:

$$\begin{cases} -\triangle u + u = f, & \text{in} \quad \Omega, \\ u - g \geq 0, \ \frac{\partial u}{\partial n} \geq 0, \quad (u-g)\frac{\partial u}{\partial n} = 0, & \text{on} \quad \partial\Omega \equiv \Gamma, \end{cases} \tag{1}$$

where n and $\frac{\partial u}{\partial n}$ denote the unit outer normal and the derivative with respect to the outward normal n on Γ, respectively, f and g are given function. Let $(\cdot,\cdot)$ denote the $L^2(\Omega)$ inner product. We introduce the convex set

$$K = \{v | \ v \in H^1(\Omega), \nu v - g \geq 0 \ \text{ a.e. on } \Gamma\},$$

where νv denotes the trace of v on the boundary Γ and in the following, we will omit the notation ν without confusion. Then corresponding a variational formulation of the problem (1) can be defined as follows:

$$\begin{cases} \text{find}\, u \in K & \text{such that} \\ a(u, v-u) & \geq (f, v-u), \forall v \in K, \end{cases} \tag{2}$$

where

$$a(u,v) = \int_\Omega (\nabla u \cdot \nabla v + uv),$$

$f \in L^2(\Omega)$ and $g \in H^1(\Omega)$.

We find that the continuous bilinear form $a(\cdot,\cdot)$ on the Hilbert space $H^1(\Omega) \times H^1(\Omega)$ satisfy

$$a(u,v) \leq \delta ||u||_{1,\Omega} ||v||_{1,\Omega}, \quad a(v,v) \geq \kappa ||v||^2_{1,\Omega}, \tag{3}$$

where δ and κ are positive constants. Then we know that the problem (2) exists a unique solution $u \in K$. Moreover, for g and f sufficiently smooth, $u \in H^2(\Omega) \cap W^{1,\infty}(\Omega)$ and the pointwise relations (1) hold (cf., [4,pp.440], [5,pp.12]).

2. Domain Decomposition

Now we shall consider domain decomposition methods with nonmatching grids. For simplicity, we assume that Ω is a bounded and convex polygomal domain in R^n. We first will divide the domain Ω into some subdomains Ω_i $(i = 1, \ldots, n_d)$ with size d_i (for simplicity $d_i = d$) and then subdivide these subdomains Ω_i and its boundary $\partial\Omega_i$ into quasi-uniform finite element meshes $T_{h_i} = \{e\}$ with size h_i and $T_{H_i} = \{\tau\}$ with size H_i, respectively. Let $h = \max\{h_i\}$ and $H = \max\{H_i\}$. We will use the following notations:

$$T^h = \cup_{i=1}^{n_d} T_{h_i}, \quad T^H = \cup_{i=1}^{n_d} T_{H_i}, \quad \Gamma_j = \Gamma \cap \partial\Omega_j \neq \phi, \ \Gamma = \cup_{j=1}^{m_d} \Gamma_j,$$

$$\Sigma = \cup_{i=1}^{n_d} \partial\Omega_i, \ \Sigma^i_{int} = \partial\Omega_i \backslash \Gamma, \qquad \Sigma_{int} = \cup_{i=1}^{n_d} \Sigma^i_{int},$$

and define the functional spaces

$$H(\Omega) = \prod_{i=1}^{n_d} H^1(\Omega_i) \ \text{ and } \ H(\Sigma) = \prod_{i=1}^{n_d} H^{-\frac{1}{2}}(\partial\Omega_i)$$

with the norm

$$|||v|||^2_{1,\Omega} = \sum_{i=1}^{n_d} ||v||^2_{1,\Omega_i} \quad \text{and} \quad |||\mu|||^2_{-\frac{1}{2},\Sigma} = \sum_{i=1}^{n_d} ||\mu||^2_{-\frac{1}{2},\partial\Omega_i}$$