COMPUTATION OF NONEQUILIBRIUM HYPERSONIC FLOW OVER CONCAVE CORNERS[*1)]

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Abstract

This paper is devoted to computation of hypersonic flow of air with chemical reactions over concave corners. A technique combining smooth transformation of domain and implicit difference methods is used to overcome numerical difficulties associated with the lack of resolution behind the shock and near the body. The implicit treatment of right hand side terms is also an important part of our method.

Key words: Shock fitting, Smooth transformation of domain, Finite difference method, Implicit method.

1. Introduction

We consider steady inviscid hypersonic flow of air about a concave corner including chemical reaction effects. Computation of nonequilibrium flow is difficult because of the steep gradients behind the shock and an entropy layer near the body. For many cases the time scale of the chemical reactions is larger than, or close to, the time scale of the original mechanical problem and the equilibrium chemistry model is not realistic. Therefore, it is necessary to evaluate the kinetics of chemistry via a nonequilibrium model.

In 1975, Rakich *et al*. [7] applied a method of characteristics to solve supersonic inviscid nonequilibrium flows. The 5-species, 18-reaction chemical model they described has since been adopted by many others [1, 2, 4]. Later, Rakich *et al*. [8] computed the flow over a concave corner. Although their method is accurate globally, it is less accurate locally. They found that the calculation for the shock angle failed to converge, resulting in a complete loss of accuracy. They were able to compute only for a short distance because of the lack of resolution near the shock. They also found that the source term resulting from chemical reactions had a strong effect on the shock solution. In 1991, Pandolfi *et al*. [4] computed the same flow introducing generalized nonequilibrium Rankine-Hugoniot (R-H) relations at the shock instead of classical R-H conditions. Using their technique, they were able to continue the computation to a very large distance. In their result details occurring over a short distance behind the shock are not given and such details are absorbed in the generalized R-H conditions.

Our goal in this paper is to design a finite difference method that will give details of the flow near the shock and compute flow accurately for a long distance. Once this is successful, we will be able to compute more difficult problems with stronger stiffness or steeper gradients for any desirable distance. We will do this using a smooth coordinate transformation under which a mesh size near boundaries represents a small physical distance while in the middle of

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the computational region it represents a large physical distance. When the mesh size in the physical domain in one direction is very small and explicit schemes are used, the mesh size in other direction must be small too. In order to make the computation efficient, we have used an implicit scheme for this problem. In this way, the computations can be done efficiently with a relatively small number of mesh points and yet the difficulty arising from lack of resolution near the shock is overcome.

2. System of Equations

The problem we consider is hypersonic flow around bodies with chemical reactions. In our chemical model of air only dissociation-recombination reactions, atom exchange reactions, and bimolecular reactions are considered. Ionization is neglected. Also the vibrational excitation of biatomic molecules is assumed half-excited so that its energy content is $RT/2$ [3]. The Euler and chemical equations are strongly coupled. We begin with the Euler equations for the steady-state configuration [4]:

$$\boldsymbol{V} \cdot \nabla \rho + \rho \nabla \cdot \boldsymbol{V} \quad = \quad 0, \tag{1}$$

$$(\boldsymbol{V} \cdot \nabla) \, \boldsymbol{V} + \frac{\nabla p}{\rho} \quad = \quad 0, \tag{2}$$

$$\boldsymbol{V} \cdot \left(\nabla h - \frac{\nabla p}{\rho} \right) \quad = \quad 0. \tag{3}$$

Here, $\boldsymbol{V}$, ρ, p and h denote the velocity vector, density, pressure and enthalpy respectively. We neglect the diffusion of the species and the equations of the production of the species along streamlines are as follows:

$$\boldsymbol{V} \cdot \nabla q_i = \omega_i, \quad i = 1, 2, 3. \tag{4}$$

Here, q_i denotes the concentration in unit $g\text{-}mole \cdot g^{-1}$ and the source term ω_i gives the rate of production for the i-th species.

With the same assumptions as in [4], we will describe the equations for a two dimensional computational frame $\{\eta, \xi\}$. We introduce a $\{z, r\}$-Descartes coordinate system and let

$$\sigma = \frac{v}{u} \quad \text{and} \quad V = \sqrt{u^2 + v^2}, \tag{5}$$

where u and v are components of velocity in z and r directions respectively. Taking p, h, σ, q_1, q_2, q_3 as dependent variables, we can have the following form of the Euler and the chemical equations under the $\{z, r\}$-Descartes coordinate system :

$$\left(1 - \frac{a^2}{u^2} \right) p_z + \sigma p_r + \rho a^2 \sigma_r \quad = \quad \frac{a^2}{u} \Psi, \tag{6}$$

$$\sigma_z + \sigma \sigma_r + \frac{p_r - \sigma p_z}{\rho u^2} \quad = \quad 0, \tag{7}$$

$$h_z - \frac{p_z}{\rho} + \sigma \left(h_r - \frac{p_r}{\rho} \right) \quad = \quad 0, \tag{8}$$

$$(q_i)_z + \sigma (q_i)_r \quad = \quad \frac{\omega_i}{u}, \quad i = 1, 2, 3. \tag{9}$$

The system is completed by the integrated form of the energy equation

$$
\begin{aligned}
H \quad &= \quad h + \frac{V^2}{2} \\
&= \quad \text{constant.}
\end{aligned}
\tag{10}
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