Journal of Computational Mathematics, Vol.18, No.6, 2000, 561–566.

ON THE ERROR ESTIMATE OF LINEAR FINITE ELEMENT APPROXIMATION TO THE ELASTIC CONTACT PROBLEM WITH CURVED CONTACT BOUNDARY*1)

Lie-heng Wang
(*State Key Laboratory of Scientific and Engineering Computing, Institute of Computational Mathematics, Academy of Mathematics and System Sciences, Chinese Academy of Sciences, Beijing 100080, China*)

Abstract

In this paper, the linear finite element approximation to the elastic contact problem with curved contact boundary is considered. The error bound $O(h^{\frac{1}{2}})$ is obtained with requirements of two times continuously differentiable for contact boundary and the usual regular triangulation, while I.Hlavacek et. al. obtained the error bound $O(h^{\frac{3}{4}})$ with requirements of three times continuously differentiable for contact boundary and extra regularities of triangulation (c.f. [2]).

Key words: Contact problem, Finite element approximation.

1. Preliminary

The error estimate of linear finite element approximation to the elastic contact problem with curved contact boundary was considered in [2], in which the authors obtained the error bound of $O(h^{\frac{3}{4}})$ with a much complex proof, requirement of three times continuously differentiable for contact boundary and extra regularities of triangulation (c. f. [2, Theorem 3.3, p.149]). In this paper, we obtained the error bound of $O(h^{\frac{1}{2}})$ with only requirement of two times continuously differentiable for contact boundary and the usual regular triangulation (c.f. [1]).

According to the notations in [2], let $\Omega = \Omega' \cup \Omega''$.

$$\mathcal{H}^1(\Omega) = \{v = (v', v'') : v' \in [H^1(\Omega')]^2, v'' \in [H^1(\Omega'')]^2\},$$

$$V = \{v \in \mathcal{H}^1(\Omega) : v' = 0 \ \text{ on } \ \Gamma_u, v''_n = 0 \ \text{ on } \ \Gamma_0\},$$

$$K = \{v \in V : v'_n + v''_n \leq 0 \ \text{ on } \ \Gamma_k\},$$

where $v_n = v_i n_i$ the normal component of the displacement, then the elastic contact problem with curved contact boundary is as follows (c.f.Fig.1):

$$\begin{cases} \text{to find} \qquad u \in K, \quad \text{such that} \\ A(u, v-u) \geq L(v-u) \qquad \forall v \in K, \end{cases} \tag{1.1}$$

* Received February 14, 1996.
1)The Project was supported by National Fundation of Sciences of China.

where

$$A(u,v) = \int_\Omega \sigma_{ij}(u)e_{ij}(v)dx,$$

$$L(v) = \int_\Omega F_i v_i dx + \int_{\Gamma_\sigma} P_i v_i ds,$$

$$e_{ij}(v) = \frac{1}{2}(\partial_j v_i + \partial_i v_j), i,j = 1,2, -\text{the tensor field of strain,}$$

$$\sigma_{ij} = c_{ijkm}e_{km}(v), i,j = 1,2, - \text{ the tensor field of stress ditermined by the generalized Hook's Law,}$$

and $c_{ijkm} = c_{jikm} = c_{kmij}$,

$$c_{ijkm}(x)e_{ij}e_{km} \geq c_0 e_{ij}e_{ij}, \tag{1.2}$$

holds for all symmetric matrices $(e_{ij})_{1\leq i,j\leq 2}$ and all $x \in \Omega$. It is well known that the equivalent boundary value problem of (1.1) is as follows (c.f.[2]):

$$-\partial_j \sigma_{ij}(u) = F_i, \qquad \text{in } \ \Omega = \Omega' \cup \Omega''; \tag{1.3}$$

$$\begin{cases} u = 0 \qquad \text{on } \ \Gamma_u, \\ \sigma_{ij}^M(u)n_j^M = P_i^M, M = ',{}'', \qquad \text{on } \ \Gamma_\sigma^M \subset \partial\Omega^M, \\ u_n = 0, T_t = 0, \qquad \text{on } \ \Gamma_0; \end{cases} \tag{1.4}$$

$$\begin{cases} u_n' + u_n'' \leq 0, \quad T_n' = T_n'' \leq 0, \\ (u_n' + u_n'')T_n' = 0, \qquad \text{on } \ \Gamma_k, \\ T_t' = T_t'' = 0, \end{cases} \tag{1.5}$$

where $T_n = \sigma_{ij}n_j n_i, T_t = \sigma_{ij}n_j t_i, n^M = (n_1^M, n_2^M)$ and $t^M = (t_1^M, t_2^M)$ are the outer unit normal and the corresponding unit tangential to $\partial\Omega^M$.

Here and what follows a repeated index always means summation over the number 1, 2.

Consider the linear finite element approximation to the problem (1.1). Let $\mathcal{T}_h'$ and $\mathcal{T}_h''$ be the regular triangulations of Ω' and Ω'' with consistency, which means that the node on Γ_k is the common node of $\mathcal{T}_h'$ and $\mathcal{T}_h''$ (c.f. Fig.2). Let V_h be the linear finite element space corresponding to V, which particularly means that $v_h' = 0$ on Γ_u and $v_{hn}'' = 0$ on Γ_0 for $v_h \in V_h$, and

$$K_h = \{v_h \in V_h : (v_{hn}' + v_{hn}'')(P) \leq 0 \qquad \forall \ \text{ nodes } \ P \in \Gamma_k\}, \tag{1.6}$$

then the linear finite element approximation to the problem (1.1) is as follows:

$$\begin{cases} \text{to find} \qquad u_h \in K_h, \qquad \text{such that} \\ A(u_h, v_h - u_h) \geq L(v_h - u_h) \qquad \forall \ \ v_h \in K_h. \end{cases} \tag{1.7}$$