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ON THE EXPLICIT COMPACT SCHEMES II: EXTENSION OF THE STCE/CE METHOD ON NONSTAGGERED GRIDS*1)

Hua-zhong Tang and Hua-mu Wu
(*State Key Laboratory of Scientific and Engineering Computing, Institute of Computational Mathematics, Chinese Academy of Sciences, Beijing 100080, China*)

Abstract

This paper continues to construct and study the explicit compact (EC) schemes for conservation laws. First, we extend STCE/SE method on non-staggered grid, which has same well resolution as one in [1], and just requires half of the computational works. Then, we consider some constructions of the EC schemes for two-dimensional conservation laws, and some 1D and 2D numerical experiments are also given.

Key words: Conservation laws, Compact scheme, Shock–capturing method, Euler equations.

1. Introduction

This paper is interested in the genuinely nonlinear conservation laws

$$\frac{\partial u}{\partial t}+\sum_{i=1}^{d}\frac{\partial f^i(u)}{\partial x^i}=0, \tag{1.1}$$

with initial data $u(0,x)=u_0(x),\ x=(x^1,...,x^d)$.

It is well known that the above problem may not always have a smooth global solution even if the initial data u_0 is adequately smooth[6]. Thus, we consider its weak solution so that the problem (1.1) might have a global solution allowing discontinuities(e.g. shock wave etc.). Moreover, the entropy condition should be imposed in order to single out a physically relevant solution(also called the entropy solution)[5, 6, 7].

In the last two decades, there has been an enormous amount of activity related to the construction and analysis of finite difference methods which approximate nonlinear hyperbolic equation (or system) of conservation laws, and are expected to have:

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(1) limit solutions which satisfy a entropy condition.

(2) the absence of spurious oscillations in the approximate solutions.

(3) at least second-order accuracy in region of smoothness, except for certain isolated points, or lines, or surfaces.

Some of the earliest work in the design of schemes having properties (2) and (3) above was done by van Leer[15], Harten[3, 4] and Sweby[10]. However, property (1) seems difficulty to prove for high-order finite difference schemes[11, 12, 17]. Here we are concerned with properties (2) and (3).

Recently Chang in [1] presented a new numerical method for 1D conservation laws, referred to as the method of space-time conservation element and solution element (STCE/SE), from a new framework. This framework differs substantially in both concept and methodology from the well-established methods e.g. finite difference, finite volume, finite element and spectral methods. But, based on his framework, it seems difficulty to be extended for multi-dimensional conservation laws.

The aim of this paper is to construct and study the explicit compact (EC) method for conservation laws based on Chang's STCE/SE framework. The paper is organized as follows. In section 2 we extend STCE/SE method to non-staggered grid for 1D conservation laws. The results show that our schemes have the same well resolution as one in [1], and just require half of computational works. In section 3 we consider construction of two classes of the EC schemes for 2D conservation laws. In section 4 some numerical experiments are given. The problems include interaction of blast waves, interaction of a moving Mach=3 shock with sine waves and regular shock reflection etc.

2. The Explicit Compact Methods for 1D Conservation Laws

Consider 1D conservation laws

$$\frac{\partial u}{\partial t} + \frac{\partial f(u)}{\partial x} = 0, \tag{2.1}$$

with initial data $u(0,x) = u_0(x)$.

As in [1], let $\xi_1 = x$, $\xi_2 = t$ be considered as the coordinates of a two dimensional Euclidean space E_2. Let Ω denote the set of mesh points (j,n) in E_2, where $n, j = 0, \pm 1, \pm 2, \cdots$, respectively. Let Ω_1 and Ω_2 denote respectively the two subsets of Ω, which are defined by

$$\Omega_1 = \{(j,n) \mid j+n \text{ is even}\}, \quad \Omega_2 = \{(j,n) \mid j+n \text{ is odd}\}.$$

Then for each $(j,n) \in \Omega_k (k = 1 \textit{ or } 2)$, there is a *solution element* $(SE_k(j,n))$ associated