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ON THE CENTRAL RELAXING SCHEMES I: SINGLE CONSERVATION LAWS*1)

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Abstract

In this first paper we present a central relaxing scheme for scalar conservation laws, based on using the local relaxation approximation. Our scheme is obtained without using linear or nonlinear Riemann solvers. A cell entropy inequality is studied for the semidiscrete central relaxing scheme, and a second order MUSCL scheme is shown to be TVD in the zero relaxation limit. The next paper will extend the central relaxing scheme to multi–dimensional systems of conservation laws in curvilinear coordinates, including numerical experiments for 1D and 2D problems.

Key words: Hyperbolic conservation laws, the relaxing scheme, TVD, cell entropy inequality.

1. Introduction

In [6], Jin and Xin constructed a class of upwind relaxing schemes for nonlinear conservation laws

$$\frac{\partial u}{\partial t} + \sum_{i=1}^{d} \frac{\partial f_i(u)}{\partial x_i} = 0, \tag{1.1}$$

with initial data $u(0,x) = u_0(x),\ x = (x_1, ..., x_d)$, by using the idea of the local relaxation approximation [2,3,6,10].

The relaxing scheme is obtained in the following way: A linear hyperbolic system with a stiff source term is first constructed to approximate the original equation (1.1) with a small dissipative correction. Then this linear hyperbolic system is solved easily by underresolved stable numerical discretizations. The main advantage of their schemes is to use neither nonlinear Riemann solvers spatially nor nonlinear system of algebraic equations solvers temporally. However, the numerical experiments have shown that

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implementation of the upwind relaxing schemes for general hyperbolic system seems to be inconvenient, because of using linear Riemann solvers of a linear hyperbolic system with a stiff source term spatially.

To overcome this drawback, we will construct a central relaxing scheme for systems of conservation laws in this series without using linear or nonlinear Riemann solvers. The schemes are shown to be TVD(total variation diminishing) and be of the similar relaxed form as in [6] in the zero relaxation limit for scalar case; a cell entropy inequality for semidiscrete schemes is also proved. Numerical experiments for 1D and 2D problems are presented in [14], which show that resolution of the central relaxing schemes is comparable to the upwind relaxing schemes presented in [6].

The paper is organized as follows. In section 2, we simply recall the relaxing system with a stiff source term, constructed by Jin and Xin to approximate Eq.(1.1). Section 3 is to construct a class of central difference approximations for the relaxing system. The schemes are also shown to have correct asymtotic limit as $\epsilon \to 0^+$, and be TVD(total variation diminishing [4]) in the zero relaxation limit in section 4. In section 5, we discuss the numerical entropy condition for the semidiscrete central relaxing scheme based on Osher–Tadmor numerical entropy flux. We conclude the paper with a few remarks in section 6.

2. Preliminaries

In this section we simply review the relaxing system with a stiff source, introduced Jin and Xin in [6] to approximate Eq.(1.1). For the sake of simplicity in the presentation, we will focus on the 1D single scalar conservation laws

$$\frac{\partial u}{\partial t} + \frac{\partial f(u)}{\partial x} = 0, \tag{2.1}$$

with initial data

$$u(0,x) = u_0(x). \tag{2.2}$$

A linear system with a stiff source term (hereafter called the *relaxing system*) can be introduced as follows:

$$\begin{aligned} &\frac{\partial u}{\partial t} + \frac{\partial v}{\partial x} = 0, \\ &\frac{\partial v}{\partial t} + a\frac{\partial u}{\partial x} = -\frac{1}{\epsilon}(v - f(u)), \end{aligned} \tag{2.3}$$

where the small positive parameter ϵ is the relaxation rate, and a is a positive constant satisfying

$$\mid f'(u) \mid \leq \sqrt{a}, \text{ for all } u \in \mathcal{R}. \tag{2.4}$$

Remark: (1) Here we can also use the more general $a(x,t)$ instead of the above constant a. The similar results can also be analyzed. (2) For scalar conservation