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NONLINEAR INTEGER PROGRAMMING AND GLOBAL OPTIMIZATION*[1)]

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Abstract

Various approaches have been developed for solving a variety of continuous global optimization problems. But up to now, less work has been devoted to solving nonlinear integer programming problems due to the inherent difficulty. This paper manages to transform the general nonlinear integer programming problem into an "equivalent" special continuous global minimization problem. Thus any effective global optimization algorithm can be used to solve nonlinear integer programming problems. This result will also promote the research on global optimization. We present an interval Branch-and-Bound algorithm. Numerical experiments show that this approach is efficient.

Key words: Integer programming, Global minimization problem, Branch-bound algorithm.

1. Introduction

Although the general linear integer programming problem is NP-hard, much work has been devoted to it (See Numhauser and Wolsey [1988], Schrijver [1986]). The solution methods include the cutting plane, the Branch-and-Bound, the dynamic programming methods etc.. However, the general nonlinear integer programming problem is difficult to solve. Garey and Johnson [1979] pointed out that the integer programming over R^n with a linear objective function and quadratic constraints is undecidable. So if a nonlinear integer programming problem is handled, it is always solved over a bounded box. Due to the inherent difficulty of nonlinear integer programming, less work has been done (see e.g. Benson, Erenguc and Horst [1990], Chichinadze [1991]). But during the past 30 years, various approaches have been developed to construct algorithms for a variety of continuous global optimization problems (for detail, see Rinooy kan and Timmer [1988]). In this paper, we transform the general nonlinear integer programming problem into an "equivalent" special continuous global minimization problem which can be solved by any one of effective global optimization algorithms. So it is a reasonable way to handle nonlinear integer programming problems. The involved functions of the considered nonlinear integer programming problem are only required to be Lipschitz

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continuous or continuous. Hence this result is a generalization of Ge [1989], where the involved functions are assumed to be twice continuously differentiable. Moreover, our proof is simple. We present an interval Branch-and-Bound algorithm for the special continuous global optimization problem. Lower bounds are calculated by the rules of interval analysis (Ratschek and Rokne [1988]). Methods for local optimal solutions can be incorporated into the Branch-and-Bound scheme to find better incumbent solutions. At last, numerical experiments are presented to show that this approach is efficient.

2. Unconstrained Case

Consider the following problem

$$(UP)_I \begin{cases} \min & f(x) \\ \text{s.t. } x \in X_I, \end{cases}$$

where $f(x) : R^n \to R$ is a Lipschitz function with Lipschitz constant L over a set X, here $X \subset R^n$ is a bounded closed box whose vertices all are integral lattices, X_I is the set of integer points in X.

A continuous global optimization problem corresponding to $(UP)_I$ is

$$(UP_\mu) \begin{cases} \min & f(x) + \mu \sum_{i=1}^{n} |\sin \pi x_i|, \; x = (x_1, \cdots, x_n)^T, \\ \text{s.t.} & x \in X. \end{cases}$$

For developing the relationship between problems $(UP)_I$ and (UP_μ), we need the following lemmas.

Lemma 2.1.

$$|\sin x| = \sin x \geq \frac{2}{\pi} x, \qquad \textit{if } 0 \leq x \leq \frac{\pi}{2}$$
$$|\sin x| = \sin x \geq \frac{2}{\pi} |\pi - x|, \qquad \textit{if } \frac{\pi}{2} \leq x \leq \pi.$$

Proof. Construct a line through points $(0, 0)$, $(\pi/2, 1)$ and a line through points $(\pi/2, 1)$, $(\pi, 0)$. Their equations are

$$y = \frac{2}{\pi} x,$$
$$y = \frac{2}{\pi} (\pi - x).$$

Since $\sin x$ is concave over $[0, \pi]$, and

$$\sin 0 = 0,$$
$$\sin \frac{\pi}{2} = 1,$$
$$\sin \pi = 0,$$