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FOURIER-CHEBYSHEV PSEUDOSPECTRAL METHOD FOR THREE-DIMENSIONAL VORTICITY EQUATION

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Abstract

In this paper, a Fourier-Chebyshev pseudospectral scheme with mixed filtering is proposed for three-dimensional vorticity equation. The generalized stability and convergence are proved. The numerical results show the advantages of this method.

Key words: Pseudospectral method, vorticity equation, error estimates.

1. Introduction

In studying boundary layers, flows past suddenly heated vertical plates and other related problems, we have to consider bilaterally periodic problems. There are several ways to solve them numerically. For instance, Murdok[1], Macaraeg[2] and Ben-yu Guo, Yue-shan Xiong[3] proposed spectral-difference schemes, while Canuto, Maday, Quarteroni[4] and Guo Ben-yu, Cao Wei Ming[5] developed spectral-finite element schemes. But the accuracy of all these schemes is still limited due to finite difference and finite element approximations, even if the genuine solution is very smooth. Therefore some authors provided various mixed spectral approximations, such as Fourier-Chebyshev approximation[6,7].

In this paper, we consider three-dimensional unsteady vorticity equation which is one of representations of incompressible flow. It possesses more unknown variables than Navier-Stokes equation and leads to non-standard boundary conditions. But in computation, this representation avoids the difficult job of constructing trial function space whose elements satisfy the incompressible condition. Thus we still use it often. We shall follow the idea of [8] to propose a mixed method by using Fourier pseudospectral approximation in periodic directions and Chebyshev pseudospectral approximation in remaining direction. This method can be implemented simply. In particular, it is easy to deal with nonlinear terms. But the pseudospectral approximation is not as stable as spectral one usually, due to the aliasing. Thus two kinds of filtering technique have been developed. The first was based on Bochner summation by Kuo Pen-yu[9,10]. The second was given by Woodward, Collela and Vandeven[11,12]. Recently, Guo Ben-yu

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improved the first one and generalized it to Chebyshev approximation[13]. The authors also developed a new mixed filtering technique for mixed approximation[8]. In this paper, we also adopt this technique and so the proposed scheme keeps the spectral accuracy, i.e., the convergence rate of infinite order.

The outline of this paper is as follows. We construct the scheme in Section 2 and present the numerical results in Section 3. The advantages of this method and the efficiency of the new mixed filtering technique are shown numerically. In Section 4, we give the main theoretical results. We list some lemmas in Section 5 and then prove the theorems in Section 6.

2. The Scheme

Let $x = (x_1, x_2, x_3)^T$ and $\Omega = I \times Q$ where $I = \{x_1 / -1 < x_1 < -1\}$, $Q = \{(x_2, x_3)/ -\pi < x_2, x_3 < \pi\}$. Let $\xi(x,t)$ and $\psi(x,t)$ be the vorticity vector and stream vector respectively with the components $\xi^{(q)}(x,t)$ and $\psi^{(q)}(x,t)$, $q = 1,2,3.\nu > 0$ is the kinetic viscosity. f_1, f_2 and ξ_0 are given functions. We consider the following problem

$$\begin{cases} \dfrac{\partial \xi}{\partial t} + J(\xi,\psi) - H(\xi,\psi) - \nu \nabla^2 \xi = f_1, & \text{in } \Omega \times (0,T], \\ -\nabla^2 \psi = \xi + f_2, & \text{in } \Omega \times (0,T], \\ \xi(x,0) = \xi_0(x), & \text{in } \Omega \bigcup \partial\Omega, \end{cases} \tag{2.1}$$

where

$$J(\xi,\psi) = [(\nabla \times \psi)\cdot \nabla]\xi, \quad H(\xi,\psi) = (\xi \cdot \nabla)(\nabla \times \psi).$$

Assume that all functions in (2.1) have the period 2π for the variables x_2 and x_3. For simplicity of the analysis, we also suppose that ξ and ψ satisfy the following boundary-value conditions as in [14],

$$\xi(\pm 1, x_2, x_3, t) = \psi(\pm 1, x_2, x_3, t) = 0. \tag{2.2}$$

The existence and uniqueness of local solution can be studied in the same way as in [14].

The inner products and norms of vector function spaces $L^2(I)$ and $L^2(Q)$ are denoted by $(\cdot,\cdot)_I, (\cdot,\cdot)_Q, \|\cdot\|_I$ and $\|\cdot\|_Q$ respectively. Let $\omega(x_1) = (1-x_1^2)^{-\frac{1}{2}}$ and define

$$(u,v)_{\omega,I} = \int_{-1}^{1} \omega u v dx_1, \quad \|v\|_{\omega,I} = (v,v)_{\omega,I}^{\frac{1}{2}},$$
$$L^2_\omega(I) = \{v/v \text{ is measurable on } I \text{ and } \|v\|_{\omega,I} < \infty\}.$$

Also define

$$(u,v)_\omega = \frac{1}{4\pi^2}\int_\Omega \omega u v dx, \quad \|v\|_\omega = (v,v)_\omega^{\frac{1}{2}}$$
$$L^2_\omega(I) = \{v/v \text{ is measurable on } \Omega \text{ and } \|v\|_\omega < \infty\}.$$