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# LONG TIME ASYMPTOTIC BEHAVIOR OF SOLUTION OF DIFFERENCE SCHEME FOR A SEMILINEAR PARABOLIC EQUATION (I)*1)

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**Abstract**

In this paper we prove that the solution of implicit difference scheme for a semilinear parabolic equation converges to the solution of difference scheme for the corresponding nonlinear stationary problem as $t \to \infty$. For the discrete solution of nonlinear parabolic problem, we get its long time asymptotic behavior which is similar to that of the continuous solution. For simplicity, we consider one-dimensional problem.

*Key words*: Asymptotic behavior, implicit difference scheme, semilinear parabolic equation.

## 1. Introduction

Let $\Omega = (0, l)$, $f(x) \in L^2(\Omega)$, $u_0(x) \in H^2(\Omega) \cap H_0^1(\Omega)$, $\phi(u) = u^3$, we consider the following initial-boundary value problem:

$$\begin{cases} \dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2} - \phi(u) + f(x) \quad \text{in} \quad \Omega \times R_+ \\ u(0,t) = u(l,t) = 0 \\ u(x,0) = u_0(x), \quad x \in \Omega. \end{cases} \tag{1.1}$$

By the usual approach[1–4] we can get the global existence of the solution of (1.1), furthermore, the solution of (1.1) converges to the solution of the following stationary problem (1.2) as $t \to \infty$.

$$\begin{cases} \dfrac{\partial^2 u}{\partial x^2} - \phi(u) + f(x) = 0 \qquad \text{in} \quad \Omega \\ u(0,t) = u(l,t) = 0. \end{cases} \tag{1.2}$$

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In this paper we prove that the solution of implicit difference scheme for (1.1) converges to the solution of difference scheme for (1.2) as $t \to \infty$.

## 2. Finite Difference Scheme

The domain $\Omega$ is divided into small segments by points $x_j = jh$ $(j = 0, 1, \cdots, J)$, where $Jh = l$, $J$ is an integer and $h$ is the space stepsize. Let $\Delta t$ be the time stepsize. For any function $w(x,t)$ we denote the values $w(jh, n\Delta t)$ by $w_j^n$ $(0 \le j \le J,\ n = 0, 1, 2, \cdots)$ and denote the discrete function $w_j^n$ $(0 \le j \le J,\ n = 0, 1, 2, \cdots)$ by $w_h^n$. We introduce the following notations:

$$\Delta_+ w_j^n = w_{j+1}^n - w_j^n \quad (0 \le j \le J-1, n = 0, 1, 2, \cdots)$$

and

$$\Delta_- w_j^n = w_j^n - w_{j-1}^n \quad (1 \le j \le J, n = 0, 1, 2, \cdots).$$

We denote the discrete function $\dfrac{\Delta_+ w_j^n}{h}$ $(0 \le j \le J-1,\ n = 0, 1, 2, \cdots)$ by $\delta w_h^n$. Similarly, the discrete function $\dfrac{\Delta_+^2 w_j^n}{h^2}$ $(0 \le j \le J-2,\ n = 0, 1, 2, \cdots)$ is denoted by $\delta^2 w_h^n$.

Denote the scalar product of two discrete functions $u_h^n$ and $v_h^m$ by

$$(u_h^n, v_h^m) = \sum_{j=0}^{J} u_j^n v_j^m h.$$

For $2 \ge k \ge 0$, define discrete norms

$$\|\delta^k w_h^n\|_p = \Big( \sum_{j=0}^{J-k} \Big| \frac{\Delta_+^k w_j^n}{h^k} \Big|^p h \Big)^{\frac{1}{p}}, \qquad +\infty > p > 1$$

and

$$\|\delta^k w_h^n\|_\infty = \max_{j=0,1,\cdots,J-k} \Big| \frac{\Delta_+^k w_j^n}{h^k} \Big|.$$

The difference equation associate with (1.1) is:

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} = \frac{\Delta_+ \Delta_- u_j^{n+1}}{h^2} - \phi(u_j^{n+1}) + f_j \tag{2.1}$$

for $j = 1, \cdots, J-1$ and $n = 1, 2, \cdots$, where $f_j = f(x_j)$.

The boundary condition of (2.1) is of the form

$$u_0^n = u_J^n = 0$$

The discrete form corresponding to (1.2) is:

$$\frac{\Delta_+ \Delta_- u_j^*}{h^2} - \phi(u_j^*) + f_j = 0, \quad 0 < j < J \tag{2.2}$$