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QUASI-INTERPOLATING OPERATORS AND THEIR APPLICATIONS IN HYPERSINGULAR INTEGRALS*1)

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Abstract

The purpose of this paper is to propose and study a class of quasi-interpolating operators in multivariate spline space $S_2^1(\Delta_{mn}^{2*})$ on non-uniform type-2 triangulation. Based on the operators, we construct cubature formula for two-dimensional hypersingular integrals. Some computing work have been done and the results are quite satisfactory.

Key words: Hypersingular integral, finite-part integral, quasi-interpolating operator, non-uniform type-2 triangulation.

1. Introduction

Since P. Zwart obtained an expression of bivariate B-spline[2], R.-H Wang and C.K. Chui have developed a series of results, especially, the quasi-interpolating operators of $S_2^1(\Delta_{mn}^2)$ on uniform type-2 triangulation and its approximation properties[1] which have widespread applications in Mechanics and Engineering. Furthermore, R-H Wang and C.K. Chui also obtained the function with minimum support in $S_2^1(\Delta_{mn}^{2*})$ on non-uniform type-2 triangulation and the basis of $S_2^1(\Delta_{mn}^{2*})$[4]. In this paper we introduce some quasi-interpolating operators of $S_2^1(\Delta_{mn}^{2*})$ on non-uniform type-2 triangulation and show their approximation properties. By using the operators we construct cubature formulas.which can be used to evaluate hypersingular integrals arisen from many mechanics and engineering problems.

2. Quasi-Interpolating Operators of $S(\Delta_{mn}^{2*})$

Let Δ_{mn}^{2*} be a non-uniform type-2 triangulation on the domain $\Omega[a,b]\otimes[c,d]$, and

$$x_{-2} < x_{-1} < a = x_0 < \cdots < x_m = b < x_{m+1} < x_{m+2},$$
$$y_{-2} < y_{-1} < c = y_0 < \cdots < y_n = d < y_{n+1} < y_{n+2}.$$

First we consider the linear operators

$$V_{mn} : C(\Omega) \to S_2^1(\Delta_{mn}^{2*}); \tag{2.1}$$

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$$V_{mn}(f) = \sum_{ij} f\Big(\frac{x_i + x_{i+1}}{2}, \quad \frac{y_j + y_{j+1}}{2}\Big) B_{ij}(x, y); \tag{2.2}$$

It is similar to the result in [1], we have the following results.

Theorem 2.1. *For $f \in P_1$ and $f = xy$, we have*

$$V_{mn}(f) = f. \tag{2.3}$$

Because of the theorem 2.7[4], we only need to verify the theorem for and $f(x, y) = x, y$ and xy. Since $V_{mn}(f)$ is a linear operator, we can only examine them in the domain D_{ij}:

$$D_{ij} = (x_i, x_{i+1}) \otimes (y_j, y_{j+1}); \quad (i = 0, \cdots m + 1; j = 0, \cdots n + 1).$$

By the computation of the values of $V_{mn}(f)$ at eight points

$$\begin{aligned}&(x_i, y_j), (x_i, y_{j+1}), \Big(\frac{x_i + x_{i+1}}{2}, y_j\Big), \Big(\frac{x_i + x_{i+1}}{2}, y_{j+1}\Big),\\ &(x_{i+1}, y_j), (x_{i+1}, y_{j+1}), \Big(x_i, \frac{y_j + y_{j+1}}{2}\Big), \Big(x_{i+1}, \frac{y_j + y_{j+1}}{2}\Big);\end{aligned} \tag{2.4}$$

we have $V_{mn}(f)$, at all of the eight points. Since the eight point are the adapt interpolating knot group in D_{ij}, we have $V_{mn}(f) = f$ in D_{ij}. Therefore, the theorem holds.

It is easy to prove that $V_{mn}(f) \neq f$, as $f = x^2$ or y^2, In order to make the theorem holds for all polynomials in P_2, we have to introduce another linear operator

$$W_{mn} : C(\Omega) \to S_2^1(\Delta_{mn}^{2*}), \tag{2.5}$$

$$W_{mn}(f) = \sum_{ij} \lambda_{ij}(f) B_{ij}(x, y), \tag{2.6}$$

where

$$\begin{aligned}\lambda_{ij}(f) =& 2f\Big(\frac{x_i + x_{i+1}}{2}, \frac{y_j + y_{j+1}}{2}\Big)\\ &- \frac{1}{4}(f(x_i, y_j) + f(x_i, y_{j+1}) + f(x_{i+1}, y_j) + f(x_{i+1}, y_{j+1})).\end{aligned} \tag{2.7}$$

It is similar to result in [4], we have the following theorem:

Theorem 2.2. *$W_{mn}(f) = f$ for any $f \in P_2$.*

By the theorem 2.1, we have $W_{mn}(f) = f$ for $f \in P_1$ and $f = xy$. Now we need to verify $W_{mn}(f) = f$ for $f(x, y) = x^2$ and y^2. Just the same as the proof of theorem 2.1, we only need to compute the values of $W_{mn}(f)$ in D_{ij} at the points

$$\begin{aligned}&(x_i, y_j), (x_i, y_{j+1}), \Big(\frac{x_i + x_{i+1}}{2}, y_j\Big), \Big(\frac{x_i + x_{i+1}}{2}, y_{j+1}\Big)\\ &(x_{i+1}, y_j),\ (x_{i+1}, y_{j+1}), \Big(x_i, \frac{y_j + y_{j+1}}{2}\Big), \Big(x_{i+1}, \frac{y_j + y_{j+1}}{2}\Big).\end{aligned} \tag{2.8}$$

By means of computation of the value of $W_{mn}(f)$, we have $W_{mn}(f) = f$ in D_{ij} for $f(x, y) = x^2$ and y^2. Therefore the theorem 2.2 holds.