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A CLASS OF ASYNCHRONOUS MATRIX MULTI-SPLITTING MULTI-PARAMETER RELAXATION ITERATIONS*1)

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Abstract

A class of asynchronous matrix multi-splitting multi-parameter relaxation methods, including the asynchronous matrix multisplitting SAOR, SSOR and SGS methods as well as the known asynchronous matrix multisplitting AOR, SOR and GS methods, etc., is proposed for solving the large sparse systems of linear equations by making use of the principle of sufficiently using the delayed information. These new methods can greatly execute the parallel computational efficiency of the MIMD-systems, and are shown to be convergent when the coefficient matrices are H-matrices. Moreover, necessary and sufficient conditions ensuring the convergence of these methods are concluded for the case that the coefficient matrices are L-matrices.

Key words: System of linear equations, asynchronous iteration, matrix multisplitting, relaxation, convergence.

1. Introduction

Multisplitting methods for getting the solution of large sparse system of linear equations

$$Ax = b, \quad A = (a_{mj}) \in L(R^n) \ \text{ nonsingular}, \quad x = (x_m), b = (b_m) \in R^n \tag{1.1}$$

are efficient parallel iterative methods which are based on several splittings of the coefficient matrix $A \in L(R^n)$. Following [1] there has been bounteous literature (see [2-11], [14-29] and references therein) on both synchronous and asynchronous parallel iterative methods in the sense of matrix multisplitting.

In this paper, based on the more recent works of [8-9] and by simultaneously taking into account of both the advantages of the matrix multisplitting and the concrete characterizations of the high-speed MIMD-systems, we further propose a class

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of asynchronous matrix multisplitting unsymmetric AOR(UAOR) methods, the multiparameter extensions of the methods given in [8] and [9], for solving the system of linear equations (1.1). These methods, besides being able to execute greatly the parallel computational efficiency of the MIMD-systems, can cover a series of practical asynchronous matrix multisplitting relaxed methods such as the novel asynchronous matrix multisplitting symmetric AOR(SAOR), unsymmetric SOR(USOR), symmetric SOR(SSOR) and symmetric Gauss-Seidel(SGS) methods as well as the known asynchronous matrix multisplitting AOR method in [8] and [9], etc., and therefore, they have certain generalities. Moreover, speedy convergence rates can be attained with suitably adjusting the relaxation parameters included in the new methods. Through some numerical results, we practically confirm that these new methods are really of distinct superiority. The convergence theories about the new methods are demonstrated in detail under more practical conditions, that is, the coefficient matrix $A \in L(R^n)$ is an H-matrix, or an L-matrix.

2. Asynchronous matrix multisplitting UAOR methods

We first split the number set $\{1, 2, \cdots, n\}$ into $\alpha(\alpha \leq n$, an integer) nonempty subsets J_i $(i = 1, 2, \cdots, \alpha)$, i.e., $J_i \subseteq \{1, 2, \cdots, n\}$ $(i = 1, 2, \cdots, \alpha)$ and $\bigcup_{i=1}^{\alpha} J_i = \{1, 2, \cdots, n\}$, where there may be overlappings among these J_i $(i = 1, 2, \cdots, \alpha)$.

For a nonsingular matrix $A = (a_{mj}) \in L(R^n)$, define matrices

$$\begin{cases} D = diag(A), \qquad \det(D) \neq 0 \\ L_i = (\mathcal{L}_{mj}^{(i)}), \quad \mathcal{L}_{mj}^{(i)} = \begin{cases} l_{mj}^{(i)}, & \text{if } j < m \text{ and } m, j \in J_i \\ 0, & \text{otherwise,} \end{cases} \\ U_i = (\mathcal{U}_{mj}^{(i)}), \quad \mathcal{U}_{mj}^{(i)} = \begin{cases} u_{mj}^{(i)}, & \text{if } j > m \text{ and } m, j \in J_i \\ 0, & \text{otherwise} \end{cases} \\ W_i = (\mathcal{W}_{mj}^{(i)}), \quad \mathcal{W}_{mj}^{(i)} = \begin{cases} 0, & \text{if } m = j \\ w_{mj}^{(i)}, & \text{otherwise} \end{cases} \\ m, j = 1, 2, \cdots, n; \quad i = 1, 2, \cdots, \alpha \end{cases}$$

such that

$$A = D - L_i - U_i - W_i, \quad i = 1, 2, \cdots, \alpha. \tag{2.1}$$

Obviously, for $i = 1, 2, \cdots, \alpha$, $L_i, U_i \in L(R^n)$ are strictly lower triangular and strictly upper triangular, respectively, while $W_i \in L(R^n)$ are zero-diagonal.

Additionally, introduce nonnegative diagonal matrices $E_i \in L(R^n)$ $(i = 1, 2, \cdots, \alpha)$,

$$E_i = diag(e_1^{(i)}, e_2^{(i)}, \cdots, e_n^{(i)}), \quad e_m^{(i)} = \begin{cases} e_m^{(i)} \geq 0, & \text{if } m \in J_i \\ 0, & \text{otherwise,} \end{cases} \tag{2.2}$$

such that $\sum_{i=1}^{\alpha} E_i = I(I \in L(R^n)$ is the identity matrix). This type of matrices $E_i(i = 1, 2, \cdots, \alpha)$ are called weighting matrices.