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FINITE ELEMENT APPROXIMATION OF EIGENVALUE PROBLEM FOR A COUPLED VIBRATION BETWEEN ACOUSTIC FIELD AND PLATE

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Abstract

We formulate a coupled vibration between plate and acoustic field in mathematically rigorous fashion. It leads to a non-standard eigenvalue problem. A finite element approximation is considered in an abstract way, and the approximate eigenvalue problem is written in an operator form by means of some Ritz projections. The order of convergence is proved based on the result of Babuška and Osborn. Some numerical example is shown for the problem for which the exact analytical solutions are calculated. The results shows that the convergence order is consistent with the one by the numerical analysis.

1. Introduction

In this paper, we study a numerical method to calculate eigen-frequencies of a coupled vibration between acoustic field and plate. A typical application of this research is to reduce a noise inside a car caused by an engine or other sources of the sound. Our study was motivated by the work of Hagiwara et al.[5]. The background of the research and some applications can be seen in [5]. We restrict our research to the problems where exact solutions can be given in a special case.

The main feature of our research is the mathematically rigorous approach to the problem. We formulate the problem as an eigenvalue problem in some Hilbert space and approximate it using the finite element method. We prove the convergence of the approximate eigenvalues. We show some numerical example for a two-dimensional test problem and check the validity of our method and analysis.

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2. Formulation of a Problem

We study the vibrations of an acoustic field coupled with a plate which consists of a part of the boundary (Fig. 1). We assume that a shape of the plate is rectangular. This

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condition together with a special boundary condition enables us to reduce the problem to a two-dimensional one (Fig. 2).

Fig. 1

Fig. 2

The time evolution problem for this coupled system is described by the following system of partial differential equations(cf. [3], where the boundary conditions are slightly different):

$$
\begin{cases}
\dfrac{\partial^2}{\partial t^2} P_0 - c^2 \nabla^2_{x,y,z}\, P_0 = 0 & \text{in} \quad \Omega_0,\ \nabla_{x,y,z} = \Big(\dfrac{\partial}{\partial x}, \dfrac{\partial}{\partial y}, \dfrac{\partial}{\partial z}\Big), \\
\qquad \dfrac{\partial P_0}{\partial n} = -\rho_0 \dfrac{\partial^2 U_0}{\partial t^2} & \text{on} \quad S_0, \\
\qquad P_0|_{\Gamma_0} = 0 & \text{on} \quad \Gamma_0, \\
\rho_1 \dfrac{\partial^2 U_0}{\partial t^2} + D \nabla^4_{y,z} U_0 = P_0|_{S_0} & \text{on} \quad S_0,\ \nabla_{y,z} = \Big(\dfrac{\partial}{\partial y}, \dfrac{\partial}{\partial z}\Big), \\
U_0|_{\partial S_0} = \dfrac{\partial^2 U_0}{\partial \sigma^2}\Big|_{\partial S_0} = 0 & \text{on} \quad \partial S_0,
\end{cases}
\tag{1}
$$

where

Ω :	two-dimensional bounded region,
$\Omega_0 = \Omega \times (0, \pi)$:	three-dimensional acoustic field,
$S_0 = S \times (0, \pi)$:	domain of plate,
$\Gamma_0 = \Gamma \times (0, \pi)$:	a part of the boundary of acoustic field, $\partial\Omega_0 = S_0 \cup \Gamma_0$,
∂S_0 :	boundary of plate,
P_0 :	acoustic pressure in Ω_0,
U_0 :	vertical plate displacement,
c :	sound velocity,
ρ_0 :	air mass density,
D :	flexural rigidity of plate,
ρ_1 :	plate mass density,
n :	outward normal vector on $\partial\Omega_0$,
σ :	outward normal vector on ∂S_0.