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AN SQP ALGORITHM WITH NONMONOTONE LINE SEARCH FOR GENERAL NONLINEAR CONSTRAINED OPTIMIZATION PROBLEM*[1]

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Abstract

In this paper, an SQP type algorithm with a new nonmonotone line search technique for general constrained optimization problems is presented. The new algorithm does not have to solve the second order correction subproblems for each iterations, but still can circumvent the so-called Maratos effect. The algorithm's global convergence and superlinear convergent rate have been proved. In addition, we can prove that, after a few iterations, correction subproblems need not be solved, so computation amount of the algorithm will be decreased much more. Numerical experiments show that the new algorithm is effective.

1. Introduction

In this paper, we consider the following optimization problem (P):

$$
\text{(P)} \qquad \begin{aligned} \min \quad & f(x) \\ \text{s.t.} \quad & g_j(x) = 0 \quad (j = 1, \cdots, m'), \\ & g_j(x) \le 0 \quad (j = m'+1, \cdots, m), \end{aligned}
$$

where $x = (x_1, \cdots, x_n)^T \in E^n$, $f(x)$, $g_j(x)$ $(j = 1, \cdots, m)$ are all real-valued smooth functions.

In recent years, Sequential Quadratic Programming (SQP) algorithms have been extensively used for the solution of such problems, and they have been widely investigated by many authors (see, e.g. [1-5]).

An attractive feature of the SQP method is that, under some suitable conditions, a superlinear convergence can be obtained, provided that the unit stepsize is eventually accepted along the direction computed by solving the quadratic programming subproblem. In order to enforce global convergence towards Kuhn-Tucker points of

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the original problem, a general approach is to define a merit function that measures progress towards the solution, and to choose a stepsize that yields a sufficient decrease in the merit function.

A standard merit function is following nondifferentiable penalty function:

$$\Phi(x) = f(x) + r\sum_{j=1}^{m'} |g_j(x)| + r\sum_{j=m'+1}^{m} \max(0, g_j(x)), \tag{1.1}$$

where r is a positive penalty parameter.

The main difficulty encountered in SQP methods with this merit function is that the line search can truncate the steplength near a solution, thus destroying the superlinear convergence. This is so-called Maratos effect. Two types of techniques have been proposed to overcome this undesirable problem. One method is the so called watchdog technique [6] in which the step of one is tentatively accepted if sufficient decrease was achieved at the previous iteration, compared to the lowest value of the merit function obtained so far. If this lowest value is not improved upon within a given finite number of iterations, the algorithm restarts from the iterate at which this value was achieved. Under some conditions it is shown that a step of one is always accepted in the vicinity of a local solution of (P). However, in the early iterations, numerous function and gradient evaluations may be wasted due to "backtracking". Another approach is that of modifying the search direction when near to a solution to avoid the Maratos effect. In this type approach, a correction subproblem must be solved at each iteration and an arc search is performed. Many additional function or gradient evaluations of constraints at auxiliary points are also needed to be performed per iteration (see, e.g., Ref. [7-8]).

In Ref.[9], by combining the arc search technique with nonmonotone line search scheme [10], Panier and Tits presented an algorithm for avoiding the Maratos effect. Their algorithm has an advantage that, after a few iterations, function evaluations are no longer performed at any auxiliary point. For early iterations, however, this algorithm still has to solve two quadratic programming subproblems to determine a search direction per step. This is not necessary for most of optimization problems.

The aim of this paper is to propose a new algorithm for improving the work of Ref.[9]. This algorithm uses a linear search scheme which is different from that of Ref.[9], and only needs to solve a quadratic programming subproblem and, if sometimes necessary, a system of linear equations at each iteration. The advantage mentioned above for the algorithm of Ref.[9] still can be preserved by the new algorithm, but the total computation amounts of new algorithm is less than that of the algorithm of Ref.[9] per iteration. Numerical experiments show that the new algorithm is very effective.

This paper is organized as follows. Algorithm A is stated in Section 2. In Section 3, under some mild assumptions, we prove that Algorithm A is global convergent. Rate of convergence is analyzed in Section 4. In Section 5, some numerical results are reported.

2. Algorithm

In the following, we let $L = \{1, \cdots, m'\}$, $M = \{m'+1, \cdots, m\}$. For any iteration point x_k, in order to compute a search direction, we will make use of a quadratic