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QP-FREE, TRUNCATED HYBRID METHODS FOR LARGE-SCALE NONLINEAR CONSTRAINED OPTIMIZATION*1)

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Abstract

In this paper, a truncated hybrid method is proposed and developed for solving sparse large-scale nonlinear programming problems. In the hybrid method, a symmetric system of linear equations, instead of the usual quadratic programming subproblems, is solved at iterative process. In order to ensure the global convergence, a method of multiplier is inserted in iterative process. A truncated solution is determined for the system of linear equations and the unconstrained subproblems are solved by the limited memory BFGS algorithm such that the hybrid algorithm is suitable to the large-scale problems. The local convergence of the hybrid algorithm is proved and some numerical tests for medium-sized truss problem are given.

1. Introduction

In this paper we consider the following nonlinear programming problem

$$\begin{aligned} &\text{minimize} && f(x) \\ &\text{subject to} && g_j(x) \geq 0, \quad j \in J = \{1, \dots, m\}. \end{aligned} \tag{1.1}$$

Extensions to problem including also equality constraints will be possible. The function $f : R^n \to R^1$ and $g_j : R^n \to R^1$, $j \in J$ are twice continuously differentiable. In particular, we apply QP-free (without quadratic programming subproblems), truncated hybrid methods for solving the large-scale nonlinear programming problems, in which the number of variables and the number of constraints in (1.1) are great. We discuss the case, where second derivatives in (1.1) are sparse and easy to be obtained.

Many iteration methods for solving (1.1) needs to solve quadratic programming (QP) subproblems at each iteration (see [?], [?], [?], [?], [?]). For large-scale case it is relative expensive. In a class of hybrid methods, proposed and developed in [?], [?], [?], [?] and [?], the subproblem is replaced with a symmetric system of not more than

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$n + m$ linear equations. In order to apply the class of hybrid methods to large-scale constrained optimization, we consider the following modifications.

(1) Instead of an exact solution, a truncated solution is determined for a system of linear equations, which is regarded as a subproblem of (1.1). This is because computing an exact solution by using a direct method such as Gaussian elimination can be expensive for large-scale problem. We research the termination criteria of the subproblem and the tradeoff between the amount of work required to compute a update direction and the accuracy with which the subproblem is solved.

(2) The conjugated gradient method is chosen as a iterative method for solving the system of linear equations.

(3) We choose the method of multiplier as a globally convergent method in hybrid method. Because the ill-conditioning can be avoided, the unconstrained subproblems are easily solved by a limited memory quasi-Newton method.

(4) In order to guarantee the numerical stability, the index set is modified and some approximated Lagrangian multipliers at each iteration are corrected such that we avoided this case where the denominators in the numerical computation is too small.

In addition, the convergence rate is proved in detail.

For the following investigation we require some notations and assumptions. The Lagrangian of problem (1.1) is defined by

$$L(x,u) = f(x) - \sum_{j=1}^{m} u_j g_j(x),$$

and its first-order and second-order derivatives with respect to the first argument are denoted by

$$\begin{aligned} \bigtriangledown_x L(x,u) &= \bigtriangledown f(x) - \sum_{j=1}^{m} u_j \bigtriangledown g_j(x), \\ \bigtriangledown_{xx}^2 L(x,u) &= \bigtriangledown^2 f(x) - \sum_{j=1}^{m} u_j \bigtriangledown^2 g_j(x), \end{aligned}$$

where $u \in R^m$ is an approximation of Lagrangian multiplier vector of (1.1). With this notation, a pair (x^*, u^*) is called a Kuhn-Tucker pair of (1.1) if the following Kuhn-Tucker conditions hold:

$$\begin{aligned} \bigtriangledown_x L(x^*,u^*) = 0, \quad u_j^* g_j(x^*) = 0, \quad j \in J, \\ u_j^* \geq 0, \quad\quad g_j(x^*) \geq 0, \quad\quad j \in J. \end{aligned}$$

The Kuhn-Tucker conditions are equivalent to the system

$$0 = \qquad P(x,u,t) = \begin{bmatrix} \bigtriangledown_x L(x,u) \\ ------- \\ t_j - g_j(x) \\ ------- \\ u_j t_j \end{bmatrix} \tag{1.2}$$

$$u_j \geq 0, \quad t_j \geq 0, \quad j \in J,$$