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CONVERGENCE OF THE POINT VORTEX METHODS FOR EULER EQUATION ON HALF PLANE*

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Abstract

In this paper, we study the point vortex method for 2-D Euler equation of incompressible flow on the half plane, and the explicit Euler's scheme is considered with the reflection method handling the boundary condition. Optimal error bounds for this fully discrete scheme are obtained.

1. Introduction

The vortex methods are efficient numerical method of simulating incompressible flow at high Reynold's number. The convergence of the vortex methods for the initial value problems of Euler equation was first obtained by Hald[4], then the results were improved by several authors[1,2,3,5]. But in fact, many practical problems are considered in a bounded domain or an exterior domain, and the numerical boundary condition has an important effect on numerical result. The particle trajectories of exact solution will not go out from the domain, but it is not the case in pratical computation. There are three kinds of method handling the boundary condition:

(a) reflection method, in which we regard the boundary as a wall, the particles will bounce back when they hit against.

(b) absorb method, in which the particles will be thrown away while they cross the boundary of the domain.

(c)extrapolation method, in which we extend the domain Ω to $\Omega' \supset \Omega$; when the particles go out from the domain Ω', they will be thrown away. But the velocity of the particles which belong to the domain $\Omega' \backslash \Omega$ will expressed through extrapolation method.

Ying Lung-an[6] proved the convergence problem of the vortex methods with extrapolation boundary treatment (c) for two dimensional bounded domains. Ying Lung-an and the author of this paper[7] got the error estimates for fully discretized two-dimensional vortex methods for initial boundary value problems of Euler equations.

To the author's knowlege, there is no convergence analysis about the other two methods. we think that the method (b) may not converge, but the method (a) may do.

For a long time, it has been widely thought the point vortex method would not converge in any finite Sobolev space. Recently, however Goodman, Hou and Lowengrub[9] have been able to prove the stability and convergence of the point vortex method for 2-D incompressible Euler equations with smooth solutions, Hou and Lowengrub[10] proved that it was also right for 3-D Euler equations.

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The convergence of the point vortex methods for initial boundary value problems of Euler equations is not available to the author's knowlege, In this paper, we will prove the convergence of the point vortex method for Euler equations on half plane handling the boundary condition with reflection method, and in consequence we can get the convergence of the vortex blob methods with the blob parameter ε which is equivalent to the grid parameter h.

Remark. The convergence of point vortex method for Euler equations on half plane cannot directly get from Goodman, Hou and Lowengrub's paper[9]. Although the problem on half plane may be considered as an initial value problem, the vorticity is not continuous, in other word, there is a vortex sheet in the solution of initial value problem, and it is important to suppose the vorticity is smooth in the convergence of point vortex method[9].

The rest of the paper is organized as follows. In section 2, we describe the point vortex method and state our major consistency, stability and convergence results for the semi-discrete point vortex method on half plane. Moreover, we prove the convergence theorem under the assumption that the consistency and stability Lemmas are valid. The consistency and stability Lemmas are proved in section 3. Finally, we consider a time discrete point vortex method in section 4.

2. Convergence of the Semi-Discrete

The 2-D incompressible inviscid Euler equations on half plane are given by

$$\frac{\partial u}{\partial t} + (u \cdot \nabla)u + \frac{1}{\rho}\nabla\pi = 0, \quad \text{in} \quad R_+^2, \tag{2.1}$$

$$\nabla \cdot u = 0, \tag{2.2}$$

$$u_2 \mid_{x_2=0} = 0, \tag{2.3}$$

$$u \mid_{t=0} = u_0(x), \tag{2.4}$$

$$\lim_{x\to\infty} u(x,t) = u_\infty,$$

where $u = (u_1, u_2)$ stands for velocity, π stands for pressure, the density ρ is a positive constant, $R_+^2 = \{x \in R^2, \mathrm{x}_2 > 0\}$, we suppose $u_\infty = 0$. $x = (x_1, x_2)$ are points in R^2, the initial data u_0 satisfies

$$\nabla \cdot u_0 = 0, u_0 \cdot n \mid_{\partial\Omega} = 0,$$

and u_0 are sufficiently smooth, then the solutions u and π are also sufficiently smooth on the domain $R_+^2 \times [0, T]$, where T is an arbitrary positive constant.

Let $\omega = -\nabla \wedge u, \omega_0 = -\nabla \wedge u_0$ and ψ be the stream function corresponding to u, then (2.1)-(2.4) is equivalent to

$$\frac{\partial \omega}{\partial t} + u \cdot \nabla\omega = 0, \quad \text{in} \quad R_+^2, \tag{2.5}$$

$$-\Delta\psi = \omega, u = \nabla \wedge \psi, \tag{2.6}$$

$$\psi \mid_{x_2=0} = 0, \tag{2.7}$$

$$\omega \mid_{t=0} = \omega_0, \tag{2.8}$$