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DOUBLE S-BREAKING CUBIC TURNING POINTS AND THEIR COMPUTATION *1)

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Abstract

In the paper we are concerned with double S-breaking cubic turning points of two-parameter nonlinear problems in the presence of $\mathcal{Z}_2$-symmetry. Three extended systems are proposed to determine double S-breaking cubic turning points. We show that there exist two kinds of singular point path passing through double S-breaking cubic turning point, One is the simple quadratic turning point path, the other is the pitchfork bifurcation point path.

1. Introduction

Many natural phenomena possess more or less exact symmetries, which are likely to be reflected in any sensible mathematical model. Idealizations such as periodic boundary conditions can produce additional symmetries. Phenomena whose models exhibit both symmetry and nonlinearity lead to problems which are challenging and rich in complexity. Problems with symmetries can show a rich bifurcation behaviour. The occurrence of multiple steady state bifurcation is mostly due to underlying symmetries. This gives rise to the difficulties to numerical computation. However, in the recent years, the tools provided by group theory and representation theory have proven to be highly effective in treating nonlinear problem involving symmetry. By these means, highly complicated situations may be decomposed into a number of simpler ones which are already understood or are at least easier to handle. In the presence of symmetries, the codimension of singularity reduces considerably and the symmetric systems have some special equivariance (see Golubitsky et al. [2], Werner and Spence[6]), which can simplify the bifurcation analysis near the multiple singular points and the numerical computation.

For the bifurcation analysis and numerical computation of double S-breaking quadratic turning points, See Werner[5], [6], also see recently Wu et al. [7] in which the authors presented a detail discussion in two-parameter dependent equations with Z_2-symmetry. However, the bifurcation analysis and numerical computation of double S-breaking cubic turning points seems to be rarely considered. The major aim of this

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paper is to present an approach for the computation of double S-breaking cubic turning points. The main idea is that, in a sense to be precise in Theorem 2.2 of Section 2, the double S-breaking cubic turning point is a simple quadratic turning point of the large extended system (1.13) provided certain conditions are satisfied. Hence, we can use the standard methods for simple turning points on this large extended system to give the required double S-breaking cubic turning points.

Consider the nonlinear problem

$$f(x,\lambda,\mu)=0,\ f: X\times R^2\to X\ , \tag{1.1}$$

where X is a Hilbert space and $f\in C^3$. We assume that f satisfies $\mathcal{Z}_2$- symmetry: there exist a linear operator $S\in L(X)$ such that $S\neq I,\ S^2=I$ and

$$Sf(x,\lambda,\mu)=f(Sx,\lambda,\mu),\quad \forall(x,\lambda,\mu)\in X\times R^2\ . \tag{1.2}$$

Then X and its dual space X' are naturally splitted into

$$X=X_s\oplus X_a,\quad X'=X'_s\oplus X'_a \tag{1.3}$$

where

$$\begin{array}{llll} X_s & =\{x\in X\mid Sx=x\}, & X_a & =\{x\in X\mid Sx=-x\},\\ X'_s & =\{\psi\in X'\mid \psi S=\psi\}, & X'_a & =\{\psi\in X'\mid \psi S=-\psi\}. \end{array}$$

It is easy to show that

$$\psi x=0,\ \text{if } (\psi,x)\in X'_s\times X_a \text{ or } (\psi,x)\in X'_a\times X_s\ . \tag{1.4}$$

We specify λ as the bifurcation parameter, and μ the auxiliary parameter. (x,λ,μ) is called a singular point of (1.1) if $f(x,\lambda,\mu)=0$ and $\dim N((f_x(x,\lambda,\mu))\geq 1$. In this paper, we are concerned with double S-breaking cubic turning point.

Definition 1.1. *A point (x_0,λ_0,μ_0) is a double S-breaking turning point of (1.1) with respect to λ if*

$$\begin{aligned} &f(x_0,\lambda_0,\mu_0)=0,\ x_0\in X_s, && (1.5a)\\ &N(f_x^\circ)=span\{\phi_1,\phi_2\},\ \phi_1\in X_s\backslash\{0\},\ \phi_2\in X_a\backslash\{0\}, && (1.5b)\\ &R(f_x^\circ)=\{x\in X\mid \psi_1 x=\psi_2 x=0\},\ \psi_1\in X'_s\backslash\{0\},\ \psi_2\in X'_a\backslash\{0\}, && (1.5c)\\ &\psi_1 f_\lambda^\circ\neq 0,\ \psi_i\phi_i\neq 0,\ i=1,2 && (1.5d) \end{aligned}$$

where $N(f_x^\circ)$ is the null space of $f_x(x_0,\lambda_0,\mu_0)$. $R(f_x^\circ)$ is the range of $f_x(x_0,\lambda_0,\mu_0)$.

A double S-breaking turning point is called a double S-breaking quadratic turning point of (1.1) if

$$D_{111}\neq 0,\ D_{122}\neq 0,\ D_{212}\neq 0. \tag{1.6}$$

A double S-breaking turning point is called a double S-breaking cubic turning point of (1.1) if

$$D_{111}=0,\ D_{122}=0,\ D_{212}=0, \tag{1.7}$$