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A MODIFIED PROJECTION AND CONTRACTION METHOD FOR A CLASS OF LINEAR COMPLEMENTARITY PROBLEMS*1)

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Abstract

Recently, we have proposed an iterative projection and contraction (PC) method for a class of linear complementarity problems (LCP)[4]. The method was showed to be globally convergent, but no statement could be made about the rate of convergence. In this paper, we develop a modified globally linearly convergent PC method for linear complementarity problems. Both the method and the convergence proofs are very simple. The method can also be used to solve some linear variational inequalities. Several computational experiments are presented to indicate that the method is surprising good for solving some known difficult problems.

1. Introduction

Let $L=\{1,\ldots,l\}$, $I\subset L$, M be an $l\times l$ positive semi-definite matrix (but not necessarily symmetric) and $q\in R^l$. For generalized linear complementarity problems

$$\text{(GLCP)}\qquad \begin{cases} u_i\ge 0, \quad (Mu+q)_i\ge 0 \quad u_i(Mu+q)_i=0, \quad \text{for} \quad i\in I\\ (Mu+q)_i=0, \qquad \text{for} \quad i\in L\setminus I, \end{cases} \tag{1}$$

we have presented a globally convergent projection and contraction method (PC method)[4]. This method is an iterative procedure which requires in each step only two matrix-vector multiplications, and performs no transformation of the matrix elements. The method therefore allows the optimal exploitation of the sparsity of the constraint matrix and may thus be efficient for large sparse problems[4]. However, only for some special GLCP's (GLCP's arising from linear programming with standard form[5,6] and from some least distance problems[8]), the improved PC methods with linear convergence are established.

In this paper, we modify the original PC algorithm in [4]. Using a new step-size rule, without the estimation of the norm of M, we are able to obtain global linear

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convergence for problem (1) in general form. Moreover, the convergence proof in this paper is much simpler than the one in [5] and [6].

Our paper is organized as follows. In Section 2, we quote some theoretical background from [4]. Section 3 describes the new algorithm and its relation to the original one. Section 4 proves the convergence properties of our new algorithm. In Section 5, we present some numerical results. Finally, in Section 6, we conclude the paper with some remarks.

We use the following notations. The i-th component of a vector u in the real l-dimensional Euclidean space R^l is denoted by u_i. A superscript such as in u^k refers to specific vectors and k usually denotes an iteration index. $P_\Omega(\cdot)$ denotes the orthogonal projection on the convex closed set Ω. $\|\cdot\|$ and $\|\cdot\|_\infty$ are the Euclidean and the max-norm, respectively. For a positive definite matrix G, the norm $\|u\|_G$ is given by $(u^TGu)^{\frac{1}{2}}$.

2. Theoretical Background

Let

$$\Omega = \{u \mid u_i \geq 0, \text{ for } i \in I\}, \tag{2}$$

$$\Omega^* := \{u \mid u \text{ is a solution of (GLCP)}\}. \tag{3}$$

Throughout the paper we assume that $\Omega^* \neq \emptyset$. The projection $v = P_\Omega(u)$ of u onto Ω is simply given by

$$v_i = \begin{cases} \max\{0, u_i\} & \text{if } i \in I, \\ u_i & \text{if } i \in L \setminus I. \end{cases}$$

It is easy to see that GLCP's can be rewritten in an equivalent way as

$$\text{(PE)} \qquad u = P_\Omega[u - (Mu + q)]. \tag{4}$$

We call it a projection equation. Based on (4) we denote

$$e(u) := u - P_\Omega[u - (Mu + q)], \tag{5}$$

and

$$\varphi(u) := e(u)^T(Mu + q). \tag{6}$$

We have the following basic lemma:

Lemma 1. *Let $u \in \Omega$, then*

$$\varphi(u) \geq \|e(u)\|^2. \tag{7}$$

A simple proof of Lemma 1 can be found in [4]. From this result we obtain immediately the following

Theorem 1. $u \in \Omega$ *and* $\varphi(u) = 0 \iff e(u) = 0 \iff u \in \Omega^*$.