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A NEW STEP-SIZE SKILL FOR SOLVING A CLASS OF NONLINEAR PROJECTION EQUATIONS*

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Abstract

In this paper, a new step-size skill for a projection and contraction method[10] for linear programming is generalized to an iterative method[22] for solving nonlinear projection equation. For linear programming, our scheme is the same as that of[10]. For complementarity problem and related problems, we give an improved algorithm by considering the new step-size skill and ALGORITHM B discussed in [22]. Numerical results are provided.

1. Introduction

In [11], an iterative projection and contraction (PC) method for linear complementarity problems was proposed. In practice, the algorithm behaves effectively, but in theory the step-size can not be proved to be bounded away from zero. So no statement can be made about the rate of convergence. Although a variant of the prime PC algorithm with constant step-size for linear programmming has a linear convergence[9], it converges much slower in parctice. In [10], He proposed a new step-size rule for the prime PC algorithm for the linear programming such that the resulting algorithm has a globally linear convergence property, and showed that the new resulting algorithm works better in practice than the prime PC algorithm. In this paper, we will introduce a new step-size skill to a projection and contraction method for nonlinear complementarity and its extensions[22]. In order to obtain this, we first make a slight modification of the prime algorithm in [22], and then give the new step-size rule. For linear programming, our ALGORITHM C discussed in this paper is the same as that of [10]. For the complementarity problem and related problems, we will give ALGORITHM D by considering ALGORITHM C in this paper and ALGORITHM B in [22]. Both in theoritical and in computational view point, ALGORITHM D is satisfactory.

Assume that the mapping $F : X \subset R^n \to R^n$ is continuous and X is a closed convex subset of R^n, we will consider the solution of the following projection equations:

$$x - P_X[x - F(x)] = 0, \tag{1.1}$$

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where for any $y \in R^n$, $P_X(y) = \text{argmin}\{x \in X | \|x - y\|\}$. (1.2)
Here $\|\cdot\|$ denotes the l_2-norm of R^n or its induced matrix norm of $R^{n\times n}$. The linear programming , nonlinear complementarity problem and nonlinear variational inequality problem can all be casted as a special case of (1.1), see [3] for a proof. For any $\beta > 0$, define

$$e_X(x, \beta) = x - P_X[x - \beta F(x)]. \tag{1.3}$$

Without causing any confusion, we will use $e(x, \beta)$ to represent $e_X(x, \beta)$. It is easy to see that x is a solution of (1.1) if and only if $e(x, \beta) = 0$ for some or any $\beta > 0$. Denote

$$X^* = \{x \in X | x \text{ is a solution of (1.1)}\}. \tag{1.4}$$

Definition 1. The mapping $F : R^n \to R^n$ is said to
(a) be monotone over a set X if

$$[F(x) - F(y)]^T(x - y) \geq 0, \quad \text{for all } x, y \in X; \tag{1.5}$$

(b) be pseudomonotone over X if

$$F(y)^T(x - y) \geq 0 \;\text{ implies } F(x)^T(x - y) \geq 0, \quad \text{for all } x, y \in X. \tag{1.6}$$

2. Basic Preliminaries

Throughout this paper , we assume that X is a nonempty convex subset of R^n and $F(x)$ is continuous over X.

Lemma 1[18]. *If $F(x)$ is continuous over a nonempty compact convex set Y, then there exists $y^* \in Y$ suth that*

$$F(y^*)^T(y - y^*) \geq 0, \quad for\ all\ y \in Y.$$

Lemma 2[23]. *For the projection operator $P_X(\cdot)$, we have*
(i)*when* $y \in X$, $[z - P_X(z)]^T[y - P_X(z)] \leq 0, \quad for\ all\ z \in R^n$; (2.1)
(ii) $\|P_X(z) - P_X(y)\| \leq \|z - y\|, \quad for\ all\ y, z \in R^n$. (2.2)

Lemma 3[2,5]. *Given $x \in R^n$ and $d \in R^n$, then the function θ defined by*

$$\theta(\beta) = \frac{\|P_X(x + \beta d) - x\|}{\beta}, \quad \beta > 0 \tag{2.3}$$

is antitone (nonincreasing).

Choose an arbitrary constant $\eta \in (0, 1)$ (e.g., $\eta = 1/2$). When $x \in X \backslash X^*$, define

$$\eta(x) = \begin{cases} \max\Big\{\eta, 1 - \dfrac{t(x)}{\|e(x,1)\|^2}\Big\}, & \text{if } \; t(x) > 0 \\ 1, & \text{otherwise} \end{cases}, \tag{2.4}$$

where $t(x) = [F(x) - F(P_X[x - F(x)])]^T e(x, 1)$.