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UNCONSTRAINED OPTIMIZATION METHODS FOR NONLINEAR COMPLEMENTARITY PROBLEM *1)

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Abstract

In this paper, we propose a class of new NCP functions and discuss their properties. By these function, we transfer the complementarity problem into unconstrained optimization problem and study the corresponding optimization problem. Numerical results are given.

1. Introduction

Since its introduction in mid-1960's and early 1970's, linear and nonlinear complementarity problems have been proven to be very useful both in optimization theories and real applications, such as the economic computation and game theoretic equilibria etc.

The standard nonlinear complementarity problem is to find a $x \in R^n$ such that:

$$F(x) \geq 0,\ x \geq 0,\ x^T F(x) = 0, \tag{1.1}$$

where $F:\ R^n \longrightarrow R^n$. For simplicity, we often call it NCP. Many authors have studied this problem and various methods for it are given. One can find an excellent summary for it in [1]. The methods mentioned in [1] are mainly based on some approximate equations and then use methods for equations to solve (1.1).

Recently, some new results about (1.1) are reported . For example, the authors of [4] consider (1.1) as unconstrained and constrained minimization. In [5], the authors propose a global Newton method for variational inequalities, which similar to the method in [4] to some extents. Both [4] and [5] transfer (1.1) into unconstrained problem which the objective function is differentiable everywhere. Kanzow[2],[3] have studied various NCP functions and give some methods, which mainly depend on the Newton equation and its local property, but some of these functions are not differentiable everywhere. For more details, see [2],[3].

In the following section, we propose a NCP function and discuss its optimal properties. In section 3, we introduce a class of functions which have the properties of the

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function in section 2. Using these functions, we transfer (1.1) into an unconstrained minimization problem, we prove that the solution of (1.1) is a global minimizer of the optimization problem, the first order and second order optimal conditions at the global minimizer are considered. Finally, numerical results of our method are given in section 4.

2. A New NCP Function

First, we give a definition which is due to [2]:

Definition 2.1. *A function $\phi : R^2 \to R$ is called NCP-function if it satisfies the nonlinear complementarity condition*

$$\phi(a,b) = 0 \Longleftrightarrow a \geq 0,\ b \geq 0,\ ab = 0.$$

Many NCP functions have been introduced and studied by different authors. There is a complete comparison for these functions in [2]. We rewrite these NCP functions as follows:

$$\begin{aligned}
I.\quad \phi(a,b) &= -ab + \frac{1}{2}\min{}^2\{0, a+b\},\\
IIa.\quad \phi(a,b) &= -ab + \min{}^2\{0, a\} + \min{}^2\{0, b\},\\
IIb.\quad \phi(a,b) &= (a-b)^2 - a|a| - b|b|,\\
IIIa.\quad \phi(a,b) &= |a-b| - a - b,\\
IIIb.\quad \phi(a,b) &= \max\{0, a-b\} - a,\\
IIIc.\quad \phi(a,b) &= \min\{a, b\},\\
IV.\quad \phi(a,b) &= \sqrt{a^2+b^2} - a - b,\\
V.\quad \phi(a,b) &= \theta(|a-b|) - \theta(a) - \theta(b),
\end{aligned}$$

where $\theta(x)$ is a strictly increasing function with $\theta(0) = 0$.

It is easy to see that, except for IV, all NCP functions mentioned above mainly use the difference between a and $|a|$, or $|a-b|$ and a,b. In fact, the functions IIa and IIb and also the ones of $IIIa - c$ are identical, except for a multiplicative constant. The functions $I, IIa - b$ are globally differentiable, $IIIa - c$ and IV is not so, the function V depends on the definition of θ. By these functions , we can transfer (1.1) into some equations and then use Newton methods to solve the equations.

In [4], through an argumented Lagrangian formulation for mathematical programming, the authors construct NCP functions as follows:

$$\phi(a,b,\alpha) = ab + \frac{1}{2\alpha}(||(-\alpha b + a)_+||^2 - a^2 + ||(-\alpha a + b)_+||^2 - b^2), \alpha > 1, \qquad (2.1)$$

where the norm is the 2-norm and $(x)_+$ denotes $(x)_+ = \max\{0, x\}$. Through this function, (1.1) is cast as an unconstrained minimization problem. The properties of the corresponding optimization problem are also disscussed.