J. At. Mol. Sci.
doi: 10.4208/jams.071713.092413a

Vol. **5** , No. 1, pp. 44-50
February 2014

Preparation of atomic entangled states and Schrödinger cat states for N trapped ions driven by frequency-modulated laser

Zhong-Jie Wang *and Xu Fang

Department of Physics Anhui Normal University, Wuhu 241000, China

Received 17 July 2013; Accepted (in revised version) 24 September 2013
Published Online 18 November 2013

Abstract. We propose a new scheme to prepare the Greenberger-Horne-Zeilinger states and atomic Schrödinger cat states for N trapped ions This scheme is based on the interaction of N trapped ions with a frequensy-modulated traveling wave light field. Preparations of these states can be all complished by one-step operation.

PACS: 42.50
Key words: trapped ion, entanglement state, atomic Schrödinger cat, frequency modulation

1 Introduction

Quantum entanglement has been enjoyed considerable attention in the last few years not only because it is fundamental in quantum mechanics but also because it plays a crucial role in quantum computation[1,2] and quantum teleportation[3].Various quantum systems have been suggested for the generation of entanglement such as trapped ions[4], nuclear magnetic resonance (NMR)[5], quantum dots[6], cavity quantum electrodynamics(CQED)[7] and others. In trapped ions system, pairs of hyperfine ground states provide an ideal host for quantum bits owing to less decoherence.. In order to entangle N trapped ions, the interaction between the ions is required and external control of this interaction is necessary to generate specific entanglement states[8]. For example, Many proposals are based on the interaction of optical Raman fields with the trapped ions[9,10,11,12]. However, these techniques require two or more laser beams acting on the trapped ions.

In this paper, we propose a scheme to prepare entanglement states and atomic Schrödinger cat states for N trapped ions using a single of frequency-nodulated laser. In our scheme, preparation of these states can be complished by one-step operation and a beam of laser is only required.

*Corresponding author. *Email address:* `wuliwzj@mail.ahnu.edu.cn` (Z. J. Wang)

2 Theoretical description

We consider N two-level ions with energy difference $\hbar\omega_\alpha$, which are trapped in a harmonic potential trap and interacts with a frequency-modulated traveling wave light field the Hamiltonian of the system can be written as $(\hbar=1)$[13]

$$\begin{aligned} H &= H_0 + H_I \\ H_0 &= \nu\alpha^\dagger\alpha + \frac{\omega_\alpha}{2}S_z \\ H_I &= \frac{\Omega}{2}\{e^{-i\omega_0 t - i\lambda\sin(\omega_p t+\varphi)}e^{ik_L x}S_+ + h.c\} \\ &= \frac{\Omega}{2}\{e^{-i\omega_0 t - i\lambda\sin(\omega_p t+\varphi)}e^{i\eta(\alpha+\alpha^+)}S_+ + h.c\} \end{aligned} \tag{1}$$

where α^+ and α are the corresponding creation and annihilation operators of center-of-mass vibrational quanta, $S_z = \sum_{j=1}^{N} S_{zj}$, $S_x = \sum_{j=1}^{N} S_{xj}$, $\sigma_{zj} = |e_j><e_j| - |g_j><g_j|$ and $\sigma_{xj} = |e_j><g_j| + |g_j><e_j|$ are Pauli operators for the j-th ion; Ω is the Rabi frequency; ω_0 is the carrier frequency, k_L wave vector; $\eta = k_L\sqrt{\hbar/2m\nu}$ is the Lamb-Dicke parameter, m mass of the ion, ν is the trapping frequency; $x = a^+ + a$ denotes a dimensionless position operator of the ion; φ is modulating phase, here we select $\varphi = \pi$; λ and ω_p is the modulating amplitude and the modulating frequency., respectively. Applying optical rotating wave approximation, we obtain

$$\begin{aligned} H_0 &= \nu\alpha^\dagger\alpha + \frac{\Delta}{2}S_z \\ H_I &= \frac{\Omega}{2}\{e^{+i\lambda\sin(\omega_p t)}e^{i\eta(\alpha+\alpha^+)}S_+ + h.c\} \\ &= \frac{\Omega}{2}\{\sum_m J_m(\lambda)e^{im\omega_p t}e^{i\eta(\alpha+\alpha^+)}S_+ + h.c\} \end{aligned} \tag{2}$$

where $\Delta = \omega_\alpha - \alpha_0$ is the detuning between the carrier frequency of light field and ionic transition frequency, $J_m(\lambda)$ is Bessel's function. In the following we select the detuning quantity $\Delta = 0$. In the interaction picture, we thus obtain

$$H_I(t) = \frac{\Omega}{2}\{\sum_m J_m(\lambda)e^{im\omega_p t}e^{i\eta(\alpha e^{-i\nu t}+\alpha^+ e^{i\nu t})}S_+ + h.c\}. \tag{3}$$

Making Lamb-Dicke approximation, selecting that the modulating frequency satisfies $\nu - \omega_p \ll \omega_p, \nu$, and neglecting the fast oscillating terms, we can obtain

$$H_I(t) = i\varepsilon_0 S_x + i\varepsilon S_x(\alpha e^{-i\delta t} - \alpha^+ e^{i\varepsilon t}) \tag{4}$$

where $\delta = v - \omega_p$, $\varepsilon_0 = \Omega J_0(\lambda)/2$, $\varepsilon = \eta\Omega J_1(\lambda)/2$. According to the definition of the displacement operator, during the infinitesimal interval $[t, t+dt]$, the corresponding evolution of