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Teleportation of a product state of an arbitrary single-particle state

Man-Lan Li [a,b,*], Liu Ye[a,*], and Jie Yang[a]

[a] *School of Physics and Material Science, Anhui University, Hefei 230039, China*
[b] *School of Sciences, Anhui Agricultural University, Hefei 230036, China*

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Abstract. This paper proposes two schemes for teleporting a product state of an arbitrary single-particle state from a sender to a receiver via a four-particle entangled cluster state. The two different quantum channels are used, while the successful probabilities of these two schemes are different. In the first proposal, the successful probability is 1.0 and in the second proposal, the successful probability is $4q^2$ if the receiver performs an appropriate unitary operation.

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Key words: quantum teleportation, probabilistic teleportation, Bell state measurement (BSM), unitary transformation

1 Introduction

Recently, much attention has been paid to quantum information. Entanglement is considered as the fundamental resource of quantum information processing such as quantum teleportation, quantum dense coding, and quantum secret sharing and so on. Quantum teleportation, first proposed by Bennett *et al.* [1] in 1993, can transmit an unknown quantum state from a sender to a receiver at a distant location via a quantum channel with the help of some classical information. As quantum teleportation is one of the basic methods of quantum communication [2] and may be useful in quantum computation [3], it has attracted much attention, and some experimental work has been reported [4, 5], much theoretical work has been reported over the past decade [6–23].

However, all of the aforementioned schemes are focused on some entangled states. In their schemes, the unknown quantum state, which is transmitted between two parties, usually is a single-particle or two-particles entangled state or three-particles entangled state, even four-particles entangled state, little attention has been devoted to teleporting a product state. In

*Corresponding author. *Email addresses:* `limanlan@ahau.edu.cn` (M. L. Li); `yeliu@ahu.edu.cn` (L. Ye)

this paper, we present two schemes for teleportating a product state via a four-particle cluster state. In the first scheme, quantum channel is a maximally cluster state; while in the second scheme, quantum channel is a non-maximally cluster state. The successful possibility of our first scheme is reach 1.0, and the successful probability of our second scheme is $4q^2$.

The rest of this paper is organized as follows. The Section 2 presents the first scheme for teleportating via a maximally cluster state. Probabilistic teleportation via a non-maximally cluster state is described in Section 3. Finally, a short conclusion is given in Section 4.

2 Teleportation of a product state of arbitrary single-particle via a four-particle cluster state

In our scheme, the two parties, a sender (namely, Alice) and a receiver (namely, Bob), Alice has a product state of arbitrary single-particle (i.e. $|\phi\rangle_{ab}=|\phi\rangle_a\otimes|\phi\rangle_b$), which she wants to send to Bob

$$|\phi\rangle_{ab}=|\phi\rangle_a\otimes|\phi\rangle_b, \tag{1}$$

where $|\phi\rangle_a=\alpha_a|0\rangle_a+\beta_a|1\rangle_a$, $|\phi\rangle_b=\alpha_b|0\rangle_b+\beta_b|1\rangle_b$. α_a, α_b, β_a and β_b are any set of complex numbers and need satisfying the following conditions: $|\alpha_a|^2+|\beta_a|^2=1$, $|\alpha_a|>|\beta_a|$, $|\alpha_b|^2+|\beta_b|^2=1$ and $|\alpha_b|>|\beta_b|$.

A cluster state is used as quantum channel between Alice and Bob, which is in the following state

$$|\varphi\rangle_{1234}=\frac{1}{2}\Big(|0000\rangle+|0011\rangle+|1100\rangle-|1111\rangle\Big)_{1234}. \tag{2}$$

Particles a, b, 2 and 3 belong to Alice; particles 1 and 4 belong to Bob. Initially, the joint system before Alice's measurement can be written as:

$$\begin{aligned}
|\psi\rangle_{ab1234}=&|\phi\rangle_a\otimes|\phi\rangle_b\otimes|\varphi\rangle_{1234}\\
=&\frac{1}{2}\Big(\alpha_a|0\rangle_a+\beta_a|1\rangle_a\Big)\Big(\alpha_b|0\rangle_b+\beta_b|1\rangle_1\Big)\Big(|0000\rangle+|0011\rangle+|1100\rangle-|1111\rangle\Big)_{1234}\\
=&\frac{1}{2}\Big(\alpha_a\alpha_b|000000\rangle+\alpha_a\alpha_b|000011\rangle+\alpha_a\alpha_b|001100\rangle-\alpha_a\alpha_b|001111\rangle\\
&+\alpha_a\beta_b|010000\rangle+\alpha_a\beta_b|010011\rangle+\alpha_a\beta_b|011100\rangle-\alpha_a\beta_b|011111\rangle\\
&+\beta_a\alpha_a|100000\rangle+\beta_a\alpha_b|100011\rangle+\beta_a\alpha_b|101100\rangle-\beta_a\alpha_b|101111\rangle\\
&+\beta_a\beta_b|110000\rangle+\beta_a\beta_b|110011\rangle+\beta_a\beta_b|111100\rangle-\beta_a\beta_b|111111\rangle\Big)_{ab1234}.
\end{aligned} \tag{3}$$

In order to realize the teleportation, twice Bell-state measurements on particles (a, 2) and particles (b, 3) are made by Alice, respectively, which will cause particles (1, 4) collapses into one of the following state

$${}_{b3}\Big\langle\phi^{\pm}\Big|_{a2}\Big\langle\phi^{+}\Big||\Psi\rangle_{ab1234}=\frac{1}{4}\Big(\alpha_a\alpha_b|00\rangle\pm\alpha_a\beta_b|01\rangle+\beta_a\alpha_b|10\rangle\mp\beta_a\beta_b|11\rangle\Big)_{14}, \tag{4}$$