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REVIEW ARTICLE

Shortcut to adiabaticity in harmonic traps

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Abstract. We review the methods to preserve the initial state populations in a fast harmonic trap expansion. The design of the time dependence of the frequency using inverse techniques presents advantages over the slow adiabatic approach, band-bang methods, or the non-local "transitionless tracking" algorithm. Many operations with cold atoms make use of adiabatic expansions and may benefit from such a shortcut to adiabaticity.

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1 Introduction

A standard operation to probe, control, or prepare a quantum system, in particular in the realm of atomic and molecular science, consists in changing the external parameters of the Hamiltonian. In many cases the ideal transformations from an initial to a final parameter configuration are the ones that do not induce any transitions [1,2]. The standard solution to this requirement is to perform the changes "adiabatically". Most experiments with cold atoms are based on a cooling stage and then an adiabatic drive of the system to some desired final trap or regime [3]. The adiabatic step may have different objectives, such as the reduction of velocity dispersion and collisional shifts for spectroscopy and atomic clocks [4], reaching extremely low temperatures unaccessible by standard cooling

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techniques [5], or, in experiments with optical lattices, broadening the state before turning on the lattice [6]. There is recently a surge of interest in adiabatic theory and applications for fundamental reasons [1,3], and also in connection with quantum information [7,8]. In adiabatic processes the system follows slowly at all times the instantaneous eigenvalues and eigenstates of the time-dependent Hamiltonian. The main problem is that the long times needed can make them useless or impractical [3], or quite simply a faster process is preferred, e.g., to increase the repetition rate of a cycle.

A highly desirable goal is to achieve the same final state as the slow adiabatic processes, possibly up to phase factors, but in a much shorter time, in other words, to find a shortcut to adiabaticity. Moreover the procedure should work for arbitrary initial states, and be realizable in practice. This goal is also relevant to optimize the passage between two thermal states of a system [9–12], a long standing question in the fields of optimal control theory and finite time thermodynamics. For time-dependent harmonic oscillators, minimal times have been established using "bang-bang" real-frequency processes believed up to now to be optimal [11], in which the frequencies are changed suddenly at certain instants but kept constant otherwise.

In this paper, we shall review different shortcuts to adiabaticity that have been proposed recently to change a harmonic Hamiltonian frequency in a finite time t_f. One approach is to design appropriate "parameter trajectories" of the frequency from the initial to the final values using bang-bang techniques [11]. Another method is to design appropriate "parameter trajectories" of the frequency based on Lewis-Riesenfeld invariants of motion [15] supplemented by simple "inverse-problem" techniques [16]. We shall call this approach "inverse-invariant" method, or II for short [13, 14]. A third approach for getting shortcuts to adiabaticity is to apply a new interaction that modifies the Hamiltonian beyond a simple parameter evolution of the original harmonic form. This option, termed "transitionless-tracking" approach, or TT for short, relies on a general framework set by Kato in a proof of the adiabatic theorem [17], and has been formulated recently by Berry [1].

We shall review first the "inverse-invariant" method in Section 2, comparing it with adiabatic and bang-bang techniques. This method will be applied to Bose-Einstein condensates governed by the Gross-Pitaevskii (GP) equation in Section 3. The TT method is reviewed in Section 4.

2 Fast optimal frictionless atom cooling in harmonic traps

2.1 "Inverse-Invariant" method

For simplicity, we shall firstly describe our method for states representing single atoms of mass m. Consider an effectively one-dimensional time-dependent harmonic oscillator,

$$H=\frac{1}{2m}\hat{p}^2+\frac{1}{2}m\omega^2(t)\hat{q}^2, \tag{1}$$