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NUMERICAL MODELING OF SHALLOW NON-NEWTONIAN FLOWS: PART I. THE 1D HORIZONTAL DAM BREAK PROBLEM REVISITED

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Abstract. The dam break problem shallow approximation for laminar flows of power-law non-Newtonian fluids is numerically revisited under a time and space second order adaptive method. Theoretical solutions are compared with experimental measurements from the literature and new ones made of silicon. Asymptotic behaviours are solved numerically and from autosimilar solutions. The obtained theoretical results are finally compared with experiments. These comparisons confirm the validity of the shallow approximation equations for non-Newtonian fluids subject to the horizontal dam break problem.

Key words. fluid mechanics; non-Newtonian fluid; power-law model; asymptotic analysis; shallow water theory

1. Introduction

Barré de Saint-Venant [27] developed in 1887 the firsts shallow water theory for fast Newtonian flows: the flow was driven by inertia terms while viscous effects were neglected. This study was first motivated by hydraulic engineering applications. More recently slower Newtonian flows [19] and the effect of viscous terms [13] were investigated. Both the manufacturing processes (concretes, foods) and the environmental applications (e.g. mud flows [11, 20], volcanic lava [14], dense snow avalanches [2] or submarine landslides [15]) require more complex non-Newtonian rheologies. For these rheologies, shallow approximations were first studied for a viscoplastic fluid by Lui and Mei [21] and revisited by Balmforth and Craster [6]. See [8, 3] for recent reviews on this subject. The dam break problem in a horizontal channel is a standard problem of fluid mechanics which finds applications in numerous environmental or industrial processes that is used as a standard benchmark for evaluating the shallow water approximations. One may also note the recent interest in a similar benchmark, the Bostwick consistometer, used in food industry [23, 22, 24, 7]. Despite these numerous applications and theoretical development, only few experimental measurements are available for the elemental case of the horizontal dam break problem. The Newtonian case has been investigated with glucose in [26] while the non-Newtonian one was discussed in [7] for power-law fluids. Furthermore, the nonlinear reduced equation obtained by the asymptotic method in the shallow limit does not admit an explicit solution and composite [18] or autosimilar solutions [17, 25, 5] were proposed instead (see also [4]). Thus, all available works are based on some simplifications and a direct numerical resolution without any simplification is of the utmost interest to fully solve this nonlinear problem, especially its long-time behaviour.

The aim of this paper is to bring a new robust and efficient numerical method for the resolution of the shallow approximation of the dam break problem and beyond, adjoin some new experimental measurements to the non-Newtonian power-law case.

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The present numerical scheme provides a fully automatic space-adaptive feature which enables an accurate capture of the front position and also is able to predict accurately the long-time behaviour of the model. Moreover, the proposed adaptive algorithm naturally extends to both viscoplastic flows and shallow approximations of three-dimensional free surface flows. The problem is solved for various power law indexes and a general front propagation rule $x_f(t)$ is proposed for any power-law index n.

This manuscript has been divided as follow: Section 2 introduces the dam break problem statement and section 3 the reduced problem obtained after the asymptotic analysis under the shallow flow approximation. Section 4 develops details of the numerical resolution of this nonlinear problem. Section 5 presents the numerical results and finally, section 6 develops the experimental measurements and the comparison between theory and experiments for the presents results and previous measurements available in the literature.

2. Problem statement

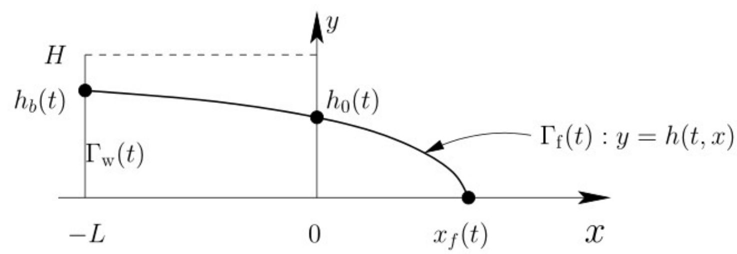

FIGURE 1. Schematic view of the dam break problem.

The classical bidimensional dam break problem for a quasi-Newtonian power-law fluid model is considered. It leads to the following problem:
find the velocity field $\mathbf{u}$ and the pressure field p, defined for all time $t > 0$ and in the time-dependent domain $\Omega(t)$, such that:

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}.\nabla\mathbf{u}\right) - \mathbf{div}\left(2\kappa\,|D(\mathbf{u})|^{n-1}\,D(\mathbf{u})\right) + \nabla p = \rho\mathbf{g} \quad \text{in }]0, +\infty[\times\Omega(t), \tag{1a}$$

$$\operatorname{div}\mathbf{u} = 0 \quad \text{in }]0, +\infty[\times\Omega(t), \tag{1b}$$

$$\mathbf{u}(t\!=\!0) = 0 \quad \text{in } \Omega(0), \tag{1c}$$

$$\mathbf{u} = 0 \quad \text{on }]0, +\infty[\times\Gamma_{\mathrm{w}}(t), \tag{1d}$$

$$\sigma.\mathbf{n} = 0 \quad \text{on }]0, +\infty[\times\Gamma_{\mathrm{f}}(t), \tag{1e}$$

where ρ is the density constant, κ the consistency constant, $n > 0$ the power-law index and $\mathbf{g}$ the gravity force vector. When $n = 1$, the fluid is Newtonian, with the classical Navier-Stokes equations. The shear thinning behaviour is associated with $0 < n < 1$ and the shear thickening behaviour to $n > 1$. The notation $|D(\mathbf{u})|$ denotes the second invariant of the rate of deformation tensor $D(\mathbf{u}) = (\nabla\mathbf{u} + \nabla\mathbf{u}^T)/2$. Here, (1a) and (1b) are respectively the momentum and mass conservation equations for an isochoric fluid. These equations are completed with the appropriate initial and boundary conditions: an initial condition for the velocity (1c), adhesion at the wall of the reservoir (1d), and equilibrium at the free surface (1e), where surface tension is neglected. Here, $\sigma = -p.I + 2\kappa\,|D(\mathbf{u})|^{n-1}\,D(\mathbf{u})$ is the total stress