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SIMULATION OF CO_2 PLUME IN POROUS MEDIA: CONSIDERATION OF CAPILLARITY AND BUOYANCY EFFECTS

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Abstract. IMplicit Pressure Explicit Saturation (IMPES) scheme with treating buoyancy and capillary forces is used to solve the two-phase water-CO_2 flow problem. In most of the previous studies of the two-phase flow, the buoyancy force term was ignored; however, in the case of liquid-gas systems such as water-CO_2, the gravity term is very important to express the buoyancy effect. In this paper, we present three numerical examples to study the CO_2 plume in homogeneous, layered, and fractured porous media. In each numerical example, we tested four different models by ignoring both gravity and capillary pressure, considering only gravity, considering only capillary pressure, and considering both gravity and capillary pressure. The cell-centered finite difference (CCFD) method is used to discretize the problem under consideration. Furthermore, we also present the stability analysis of the IMPES scheme. The numerical results demonstrate the effects of the gravity and the capillary pressure on the flow for the four different cases.

Key words. IMPES, stability, carbon dioxide (CO_2) sequestration, two-phase flow, heterogeneous porous media, capillary pressure.

1. Introduction

Carbon dioxide (CO_2) sequestration is one of the most effective methods to reduce the amount of CO_2 in the atmosphere by injecting CO_2 into the geological formations. Furthermore, it has also an important role for the enhanced oil recovery [3, 9, 25, 30, 31]. Numerical investigation of the two-phase flow in porous media with the application on the CO_2 injection has been studied by several authors in the recent years [7, 9, 16, 23, 29, 30]. A coupled system of the time-dependent partial differential equations is used to govern the two-phase flow in porous media. There are several methods to solve these time-dependent partial differential equations such as the fully implicit, IMplicit-EXplicit (IMEX), operator splitting, sequential, and IMplicit Pressure Explicit Saturation (IMPES). The latter is the most well-known method and widely used in the subsurface modeling and simulation. The fully implicit method [6, 15, 26–28] is unconditionally stable; however, it is computationally expensive. The IMEX method [2, 5, 21] is used to solve the ordinary differential equations resulting from the spatial discretization of the time-dependent partial differential equations. The IMEX method is more stable than the fully implicit method because it considers the linear terms implicitly and solves the other terms explicitly. The idea of the operator splitting method [1, 17, 20] is to simplify the original problem into a simpler form by the time-lag dimension. Some authors [20, 22] introduced the iterative operator splitting as a part of the step of iteration in the fully implicit method. The sequential method [33] is a modified version of the IMPES method since the saturation is evaluated implicitly too. Comparing with the fully implicit method, the computational cost of the sequential method is not so expensive in such a way that it is suitable for the large size models where the stability becomes an important consideration.

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In the modeling of the two-phase incompressible flow in porous media, the most well-known method that is employed is the IMplicit Pressure Explicit Saturation (IMPES) [11–14, 18, 24, 26, 28]. In this scheme, the pressure equation is evaluated implicitly while the saturation equation is solved explicitly. The pressure equation in the IMPES scheme is obtained after summation of the Darcy's law and substitutes them into the summation of the two mass conservation equations of each phase. After obtaining the pressure, we can calculate the Darcy velocity and the saturation. Since the pressure does not depend on the time, the pressure changes slower than the saturation ones. The implicit part of the pressure calculation takes more time than the explicit part of the saturation computation. Basically, the IMPES method is the fastest scheme based on the per-time step basis. Moreover, the stability condition of the scheme is determined in terms of the given parameters.

From some studies that have been done, from the physical point of view, there is a significant difference between the results considering the capillarity and without the capillarity [4]. The capillary pressures hold up the buoyant flow of the CO_2 plume. Moreover, the capillary pressure together with the relative permeability governs the interactions between the medium and the fluid path. It, however, could be neglected in homogeneous domain but not for heterogeneous one [24]. Therefore, it is important to include the capillarity effects simultaneously with the gravity in this work.

The paper contains several parts: Section 1, in general, is an overview about the motivation and the importance of CO_2 sequestration and its numerical investigation. In section 2, we describe the governing equations for the two-phase flow in porous media for the water-gas system. In section 3, we present the IMPES numerical scheme including both the gravity and the capillary pressure terms which is constructed based on the cell-centered finite difference (CCFD) method, explanation on how to generate the non-uniform mesh, and the treatment of boundary conditions. In section 4, we discuss the stability of the IMPES scheme. Section 5 demonstrates some numerical experiments of the CO_2 injection in the subsurface and the discussion of the results. Finally, we end up with the conclusions in section 6.

2. Mathematical formulations

The basic differential equations that govern the flow of the two-phase incompressible flow in porous media are the saturation equation and the constitutive equation (Darcy's law). Those equations predict the fluid pressure distribution, the velocity, and the phase saturation. In this work, we consider the liquid (water) and the gas (CO_2) fluid phases and we assume there are no effects of the solubility of CO_2 in the water and the salt precipitation. The water phase is indicated by a subscript w and the gas phase is denoted by g. The saturation equation in this paper is defined as follows [23]:

$$\phi \frac{\partial S_\alpha}{\partial t} + \nabla \cdot \mathbf{u}_\alpha = q_\alpha, \ \alpha = w, g, \tag{1}$$

where ϕ is the porosity of the medium, S_α, $\mathbf{u}_\alpha$, and q_α are the saturation, the Darcy velocity, and the source/sink of each phase α, respectively. In the standard notations, $\nabla \cdot (\frac{\partial}{\partial x}, \frac{\partial}{\partial z})^T$ is the partial differential operator where $\nabla = (\frac{\partial}{\partial x}, \frac{\partial}{\partial z})^T$ is the gradient operator and $\nabla \cdot$ is the divergence operator.

The fluid saturations for the two-phase flow of the water and the gas are interrelated by,

$$S_w + S_g = 1 \tag{2}$$