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ADAPTIVE ASYMPTOTIC STABILIZATION OF A BIOPROCESS MODEL WITH UNKNOWN KINETICS

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Abstract. The stability of a four-dimensional nonlinear model of a wastewater treatment process (proposed by Bernard, Hadj-Sadok, Dochain, Genovesi and Steyer in 2001) is studied. A feedback control law, depending only on on-line measurable quantities, is proposed. This feedback law stabilizes asymptotically the closed-loop system towards a previously chosen operating point. In order to prove that the closed-loop system is asymptotically stable, two Lyapunov-like functions are constructed explicitly. A numerical extremum seeking algorithm is then applied to stabilize the dynamics towards an equilibrium point corresponding to the maximum methane output flow rate. The robustness of the feedback is demonstrated by involving uncertainties in the growth rate model functions. Computer simulations are reported to illustrate the theoretical results.

Key words. a model of a wastewater treatment process, adaptive feedback, asymptotic stability, extremum seeking

1. Introduction

Biological wastewater treatment using anaerobic digestion is a process, where microorganisms decompose the organic compounds released in industrial and urban effluents (plant residues, animal wastes, food industry wastewater, etc.). The main goal is to reduce the pollutant concentration in the outlet stream below a specified value, usually fixed by environmental and safety rules. At the same time this process can become profitable with the production of valuable energy (methane). The operation of such processes poses a number of practical problems, since anaerobic digestion is known to be highly unstable under variation of the operating conditions, see e. g. [2], [19] and the references therein.

In the recent years the dynamic modeling of anaerobic digestion has become an active research area. This is due to the fact that a mathematical model of the plant can be used as a powerful tool to simulate different operating, control and optimization strategies. The design of such models should find a "trade-off" between model complexity and mathematical investigation of the model, especially for control purposes [14].

One of the main drawbacks in the modeling and control of the anaerobic digestion lies in the difficulty to monitor on-line the key biological variables of the process and in estimating the expressions of the bacterial growth rates. Thus developing control systems only based on simple measurements and minimal assumptions on the growth rates that guarantee stability of the process is of primary importance ([2], [16], [19] and the references therein).

The present paper is devoted to studying a known four-dimensional nonlinear control system, that models a wastewater treatment process [1], [2], [6], [7], [14], [22]. In a previous work [10] the authors design an adaptive stabilizing feedback control law for the same model in the presence of parameter uncertainties. This adaptive feedback depends on the observable state variables s_1 and x_1 (see the

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definitions in the next section) and stabilizes asymptotically the closed-loop system towards an equilibrium point such that its projection on the s_1-axis is equal to a previously chosen operating point s_1^*. Unfortunately, the variable x_1 is not on-line measurable. For that reason, here we propose an adaptive feedback law for asymptotic stabilization, that is more practically oriented, i. e. it depends only on on-line measurable quantities, the so-called biological oxygen demand. A model-based numerical extremum seeking algorithm [11] is then applied to stabilize the closed-loop system to the equilibrium point with maximal production of methane. Optimization via extremum (peek) seeking is recently an extensively used approach in optimization the productivity of a continuously stirred tank bioreactor. In the literature (see [3], [20], [23], [24], [25]) the extremum seeking approach is not model-based: the algorithm is usually presented in the form of a block-scheme (diagram) to iteratively adjust the dilution rate directly in the bioreactor in order to steer the process to a point, where optimal value of the output is achieved. The main restriction in applying this model-free extremum seeking approach is that the dynamics should be open-loop stable. Otherwise, a locally stabilizing controller is needed to stabilize the equilibrium points around the optimal operating point. Our approach is different: we first globally stabilize the dynamics towards a given equilibrium and then apply the numerical extremum seeking algorithm to drive the system to the desired state.

The paper is organized as follows. Section 2 presents shortly the dynamic model of the wastewater treatment process. The asymptotic stabilization of the dynamic system towards a previously chosen operating point (called also reference or set point) is studied in Section 3 under general assumptions on the growth rates. In order to prove that the closed-loop system is asymptotically stable, suitable Lyapunov-like functions are constructed explicitly. Choosing in a proper way different operating points, the extremum seeking algorithm [11] is used in Section 4 to stabilize the dynamic system towards the equilibrium point where the maximum production of biogas (methane) is achieved. Computer simulations illustrating the theoretical results, are reported in Section 5. To demonstrate the robustness of the feedback, we assume that the bacterial growth rate functions are not exactly known but involve uncertainties. For convenience of the reader the main steps of the extremum seeking algorithm are sketched in the Appendix.

2. Model description

We consider a model of an anaerobic digestion process, based on two main reactions [7], [14], [15]: (a) acidogenesis, where the organic substrate (denoted by s_1) is degraded into volatile fatty acids (VFA, denoted by s_2) by acidogenic bacteria (x_1); (b) methanogenesis, where VFA are degraded into methane CH_4 and carbon dioxide CO_2 by methanogenic bacteria (x_2). We assume that the methane flow rate is the measurable output and denote it by Q. The mass balance model of a continuously stirred tank bioreactor is described by the following nonlinear system