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LOCAL VELOCITY POSTPROCESSING FOR MULTIPOINT FLUX METHODS ON GENERAL HEXAHEDRA

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Abstract. The authors formulated in [32] a multipoint flux mixed finite element method that reduces to a cell-centered pressure system on general quadrilaterals and hexahedra for elliptic equations arising in subsurface flow problems. In addition they showed that a special quadrature rule yields $\mathcal{O}(h)$ convergence for face fluxes on distorted hexahedra. Here a first order local velocity postprocessing procedure using these face fluxes is developed and analyzed. The algorithm involves solving a 3×3 system on each element and utilizes an enhanced mixed finite element space introduced by Falk, Gatto, and Monk [18]. Computational results verifying the theory are demonstrated.

Key Words. mixed finite element, multipoint flux approximation, cell-centered finite difference, mimetic finite difference, full tensor coefficient, quadrilaterals, hexahedra, postprocessing.

1. Introduction

A major motivation for defining accurate locally conservative numerical methods for elliptic equations with tensor coefficients is the increasing interest in the modeling of subsurface flow and transport in porous media. Subsurface systems or geosystems may be natural, such as aquifers and fossil fuel reservoirs, or artificial, such as landfills and nuclear waste sites and are seen today as resources that must be managed. Geosystems are complex, however, for they involve multiple physical and chemical processes operating across multiple spans of time (from nanoseconds to centuries) and space (from nanometers to kilometers) and involve highly varying heterogeneities.

Effective management of a geosystem must be based on conceptual and numerical models of the geosystem. An important example of geosystem applications is CO_2 sequestration, which is the long-term isolation of carbon dioxide from the atmosphere in geological reservoirs. Geologic sequestration by injection of CO_2 into deep brine aquifers and reservoirs represents one of the most promising approaches for reducing the increases in atmospheric CO_2, which have been blamed for recent trends in global warming and alarming changes in weather patterns. The basis for this potential is the huge global storage capacity existing in geologic formations and the availability and close proximity of potential injection sites to power generation

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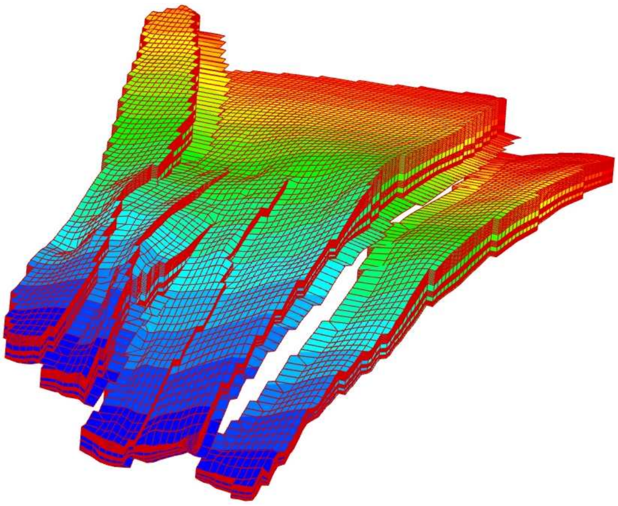

FIGURE 1. Hexahedral meshes in Frio CO_2 demonstration site

plants. Another example is the disposal of nuclear wastes. The safe disposal of nuclear waste in geologic media is a complex problem that requires extensive modeling and simulation to assess the long-term performance of the disposal system. The calculations have to address the response of the site over thousands of years and incorporate multiscale and multiphysics coupling to various extents depending on the geologic medium. Predictive computational simulation is essential for providing the information needed to make decisions on site selection, design, and operation of repositories long before the repository response can be measured. In addition, uncertainty quantification will play a major part in the modeling and simulation of the repository response. Other examples include methane gas migration, bioremediation, management of groundwater systems, geothermal systems, increasing oil and gas production, and CO_2 injection for enhanced oil and gas recovery.

Although each geosystem mentioned above has its unique physics that require site-specific models, all geosystem models will have at their base certain general capabilities to which site-specific capabilities can be added. These general capabilities include multiscale and multiphysics models and numerical algorithms for approximating the pertinent physical, chemical, geological, and biological processes characteristic of these systems. Effective modeling of geosystems necessitates the formulation of accurate and efficient locally conservative algorithms for computing velocities and pressures on general grids [16, 13]. Using hexahedra involves fewer degrees of freedom than tetrahedral grids and can accurately represent geological layers as shown in Figure 1, the Frio CO_2 demonstration site. In addition, as discussed above, geosystem models involve modeling different processes such as diffusion/dispersion and reactive transport and thus requiring accurate velocities within elements and on faces. Examples include parabolic equations, streamline methods, and using higher order discontinuous Galerkin approximations for transport.

Here we consider multipoint flux mixed finite element (MFMFE) discretizations for Darcy flow on general hexahedral grids. The method is motivated and closely related to the multipoint flux approximation (MPFA) method [1, 2, 15, 14]. In the MPFA finite volume formulation, sub-edge (sub-face) fluxes are introduced, which allows for local flux elimination and reduction to a cell-centered scheme.