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DOWNSCALING: A COMPLEMENT TO HOMOGENIZATION

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Abstract. A groundwater flow model based on a specified hydraulic conductivity field in the modeling domain has a unique solution only if either the head or the normal flux component is specified on the boundary. On the other hand, specification of both head and flux as boundary conditions may be used to determine the conductivity field, or at least improve an initial estimate of it. The specified head and flux data may be obtained from measurements on the boundary, including the wells. We have presented a relatively simple, but instructive approach: the Double Constraint (DC) method. The method is exemplified in the context of upscaling and its inverse: downscaling. The DC method is not only instructive, but also easy to implement because it is based on existing groundwater modeling software. The exemplifications shown in this paper relate to downscaling and demonstrate that the DC method has practical relevance.

Key Words. Double Constraint Method, Downscaling, Inverse problems, Conductivity

1. Introduction

The Double Constraint (DC) method is a relatively simple, yet very instructive approach to inverse modeling. In this paper the DC method has been applied to downscaling, which can be considered as a practical complement to upscaling. However, the DC method is applicable in a wider range of settings, especially in applications in which wells play a role. The DC method is instructive, because it shows all the ingredients required for inverse modeling: measured heads and fluxes at the same location on the closed boundary, as well as estimated conductivities — the priors. At the same time, the method can be easily implemented, provided that groundwater modeling software is available.

In the context of groundwater flow, a forward model is a model in which the hydraulic conductivity is specified everywhere in the modeling domain. A forward model has a unique solution provided that appropriate boundary conditions are imposed. Considering groundwater flow this is the case only if either the head on a part of the boundary of the modeling domain, or the flux through that part of the boundary is specified in any point. Specification of both head and flux at that part of the boundary over-specifies the problem and has, therefore, no solution. However, such an over-specification may be used to improve the initially estimated conductivity field by conditioning it to the measured hydraulic data head and flux, in such a way that downscaling is meaningful. Determination of conductivities from additional boundary data is generally called inverse modeling. In our approach we follow the main steps of a method that has proved its applicability in Eletrical Impedance Tomography, [1, 4, 6, 9].

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After an introduction to downscaling in section 2, the double constraint method is presented in section 3. An exemplification of downscaling for a grid block far removed from wells is shown in section 4, where two isotropization equations — Wexler's equation and the square root equation — have been compared. A similar example is briefly presented in section 5. Section 6 presents a summary, conclusions and discussion, while section 7 shows the references.

For reasons of simplification, 2-dimensional problems will be considered. Extension to 3D problems is straightforward.

2. Downscaling

In this paper downscaling is considered as a practical complement to upscaling with application in groundwater flow modeling.

Upscaling starts with a fine-scale model with heterogeneous fine-scale conductivities in the elements (triangles, grid blocks) of a gridded rectangular upscaling cell. From these fine-scale conductivities the homogeneous effective coarse-scale conductivity of the upscaling cell is determined. A variety of upscaling methods has been applied and published, starting from well-known arithmetic and harmonic averages for flow respectively parallel and normal to layers, as well as the geometric average for fine-scale isotropic checkerboard patterns. For more complex fine-scale conductivity configurations the renormalization method can be applied, or a large class of methods based on fine-scale solution of the flow equation - see [7] for a review. Then based on specific discharge rates and head gradients in the fine-scale elements, the upscaled conductivity may be computed.

With respect to the latter class of methods, the question of boundary conditions to impose on the upscaling cell arises. Homogenization, probably the most popular method from this class, assumes periodicity of the porous medium and, as a consequence, periodic boundary conditions. Presumably, boundary conditions that are consistent with the actual flow might appear superior above the more-or-less arbitrarily chosen periodic boundary conditions. However, when using boundary conditions derived from an actual flow pattern, there is no consistency between different possible definitions of a large-scale conductivity, [11]. It should be mentioned that there exists another category of methods capable of dealing directly with a multiscale structure of the medium. A wide overview of such methods is given in [5]; this topic will not be further addressed in this paper.

A coarse-scale model consists of grid blocks (in a finite difference setting) in which each grid block has a coarse-scale conductivity that is obtained by upscaling from fine-scale conductivities. Once the solution of the flow problem in the large scale is computed, the modeler (the geohydrologist) may want to zoom in into the details of the groundwater flow in one or more coarse-scale grid blocks. If the original fine-scale conductivity distribution — from which the coarse-scale conductivity was derived by homogenization — is still known, we can run a fine-scale flow model on one large-scale cell with boundary conditions derived from the flow pattern calculated by the coarse-scale model. The fine-scale boundary conditions should be such that: (i) the total inflow through the boundary of the fine-scale model should be equal to the inflow calculated by the coarse-scale model, and (ii) the average head on each boundary node of the fine-scale model should be equal to the average head calculated by the coarse-scale model. Also wells may be considered as boundaries.