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REDUCED-ORDER MODELING OF BURGERS EQUATIONS BASED ON CENTROIDAL VORONOI TESSELLATION

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Dedicated to Professor Max D. Gunzburger on the occasion of his 60th birthday

Abstract. In this paper, we study a reduced-order modeling for Burgers equations. Review of the CVT(centroidal Voronoi tessellation) approaches to reduced-order bases are provided. In CVT-reduced order modelling, we start with a snapshot set just as is done in a POD(Proper Orthogonal Decomposition)-based setting. We shall investigate the technique of CVT as a procedure to determine the basis elements for the approximating subspaces. Some numerical experiments including comparison of CVT-based algorithm with numerical results obtained from FEM(finite element method) and POD-based algorithm are reported. Finally, we apply CVT-based reduced order modeling technique to a feedback control problem for Burgers equation.

Key Words. reduced-order modeling, proper orthogonal decomposition, centroidal Voronoi tessellations, Burgers equations.

1. Introduction

In the computational simulation of (nonlinear) complex, turbulent, or chaotic systems, standard discretization schemes (finite element, finite difference, etc.) may require many thousands or even millions of degrees of freedom for the accurate simulation of fluid flows. As a result, these schemes for the spatial discretization lead to sparse but very large-scale, and in general, nonlinear systems of ordinary differential equations (ODEs) and the approximate solutions using these approaches are expensive with respect to both storage and computing time. The situation is even worse for optimization problems for which multiple solutions of the complex state system are usually required or in feedback control problems for which real-time solutions of the complex state system are needed.

In order to overcome this difficulty, reduced-order modeling was introduced. Roughly speaking, this technique is to replace a given mathematical model of a system or process by a model that is much “ small ” than the original model, but still describes (at least approximately) certain aspects of the system or process. That is to say, the types of reduced-order models are those that attempt to determine accurate approximate solutions of a complex system using very few degrees

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of freedom. To do so, such models have to use basis functions that are in some way intimately connected to the problem being approximated. This technique is based on projecting the dynamical system onto subspaces consisting of basis elements that contain characteristics of the expected solution. This is in contrast to the traditional numerical methods such as finite difference method (FDM) which uses grid functions as basis functions or finite elements method (FEM) which uses piecewise polynomials for this purpose.

The ideas underlying the reduced-basis method appear to have their origins in the suggestions of Almroth [1] and Nagy [13], which were developed by Noor and colleagues [14]-[16] in the context of simulations for structures and later by Peterson [18] in high Reynolds numbers incompressible viscous flow simulations. Roughly speaking, the reduced-basis method employs parameter-dependent solutions of the system to be approximated. These solutions are used to construct basis elements in the hope that solutions at other parameter values can be represented in terms of perturbations of solutions given at carefully chosen parameter values (the Lagrange basis approach) or in terms of a "moving frame" (the Taylor approach). It is important to note that the parameter-dependent solutions used as basis functions can be obtained either from full-order model numerical simulations or experimental data.

In this article, we focus on the *centroidal Voronoi tessellation* (CVT) as reduced order modelling technique which is currently an active research field. Centroidal Voronoi tessellation-based reduced-order modeling of fluid flows was developed by [8, 9]. In CVT-reduced order modelling, we start with a snapshot set just as is done in a POD-based setting. However, instead of determining a POD basis from the snapshot set, we apply our CVT methodologies to determine the generators of a CVT of the snapshot set; these generators constitute the reduced-order basis. We then use the CVT-based basis in just the same way as one uses a POD-based basis to determine a very low-dimensional approximation to the solution of a complex system. CVT also possesses an optimality property, although it is different from that possessed by POD bases. In this article, we shall investigate the CVT method as a reduced order model for the unsteady Burgers equation with appropriate initial and boundary conditions. As a matter of fact, POD and CVT may be viewed as simply different procedures to determine the basis elements for the approximating subspaces. We shall investigate the technique of CVT as a procedure to determine the basis elements for the approximating subspaces.

The plan for the rest of paper is as follows. In section 2, we give some definitions and property of CVT's, and two approaches for computing these tessellations. Section 3 is devoted to applying CVT to solve the time-dependent Burgers equation and, some numerical experiments including comparison of CVT-based algorithm with numerical results obtained from FEM and POD-based algorithm are reported in section 4. Finally, we apply CVT-based reduced-order modeling technique to a feedback control problem for Burgers equation in section 5.

2. Centroidal Voronoi Tessellation

The concept of the *centroidal Voronoi tessellations* (CVTs) has been studied in [5]. CVTs have been successfully used in several data compression settings, e.g.,