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Reconstruction of Small Inclusions in Electrical Impedance Tomography Problems

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Abstract. An inverse problem to recover small inclusions inside a two-layer structure is considered. Integral representations for the solution of two-layer inhomogeneous conductivity problem are derived and asymptotic expansions of a perturbed electrical field are obtained. Moreover, the uniqueness of the recovery of the locations and conductivities of small inclusions is proved.

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Key words: Electrical impedance tomography, small inclusion, inverse problem, uniqueness.

1. Introduction

Electrical impedance tomography (EIT) is a noninvasive type of medical imaging, where the electrical conductivity of a part of the body (inclusion) is inferred from surface electrode measurements. The current sources — e.g. electrodes, are placed on the surface and voltage is measured (or vice versa) at a few or all source positions. EIT has been widely used in monitoring the lung function [15], breast cancer imaging [4, 19] and so on. In mathematical terms, the problem consists in the recovering of conductivity from surface currents and potentials. The unique recovering from infinitely many measurements or from the Newmann-to-Dirichlet map is considered in Refs. [5, 7, 13, 16, 20]. If only finitely many measurements are available, the uniqueness is related to recovery of the inclusion shape, whereas global uniqueness is only obtained for convex polyhedrons and balls in three-dimensional space and for polygons and disks in the plane — cf. [6, 10–12, 14, 18].

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We also refer the reader to works [8,9,21], where numerical methods in imaging are considered. Here, we deal with the recovery of small inclusions inside of an inhomogeneous body, containing inhomogeneous core and homogeneous outer layers. The small inclusions are buried in the homogeneous layer and we only need to use one measurement to recover the corresponding locations and conductivity.

Let D be a central region generating an electric field and $\Omega\setminus\overline{D}$ be the surrounding body tissue. We assume that D is compactly supported in Ω — i.e. $D \subset\subset \Omega$, and consider the following problem:

$$\begin{aligned} &\nabla\cdot\sigma\nabla u = f \quad \text{in} \quad \mathbb{R}^3, \\ &u(\mathbf{x}) = \mathscr{O}(|\mathbf{x}|^{-1}) \quad \text{as} \quad \mathbf{x}\to\infty \end{aligned} \tag{1.1}$$

with the source term f supported in D and such that

$$\int_{\mathbb{R}^3} f(\mathbf{x})d\mathbf{x} = 0.$$

If there are no small inclusions, the electric conductivity function has the form

$$\sigma(\mathbf{x}) = \sigma_0(\mathbf{x})\chi(D) + k_0\chi(\Omega\setminus\overline{D}) + \chi(\mathbb{R}^3\setminus\overline{\Omega}), \tag{1.2}$$

where χ denotes the characteristic function. Besides, we also assume that the background tissue contains a group of small inclusions. Let $l_0 \in \mathbb{N}^+$ and D_l be small inclusions with the conductivities $k_l > 0$, $l = 1,2,\cdots,l_0$. In the presence of small inclusions $(D_l;k_l)$, $l = 1,2,\cdots,l_0$, the conductivity distribution can be described as

$$\sigma(\mathbf{x}) = \sigma_0(\mathbf{x})\chi(D) + k_0\chi(\tilde{D}) + \sum_{l=1}^{l_0} k_l\chi(D_l) + \chi(\mathbb{R}^3\setminus\overline{\Omega}), \tag{1.3}$$

where

$$\tilde{D} := \Omega\setminus\overline{\bigcup_{l=1}^{l_0} D_l \cup D}.$$

Let u_0 and u be, respectively, the solutions of the conductivity problems (1.1), (1.2) and (1.1), (1.3). Here, we are looking for the solution of the following inverse problem:

$$(u(\mathbf{x}) - u_0(\mathbf{x}))\Big|_{\mathbf{x}\in\partial\Omega} \longrightarrow \bigcup_{l=1}^{l_0}(D_l;k_l).$$

More exactly, we want to recover small inclusions by monitoring the change of the electric field on the boundary $\partial\Omega$. Assume that

$$D_l = \delta B + \mathbf{z}_l, \quad l = 1,2,\cdots,l_0, \tag{1.4}$$

where $\delta << 1$ and B is a simply-connected $C^{1,1}$ domain centered at the origin.

In this work we use asymptotic analysis, the layer potential technique and the unique continuation theorem to show the possibility of the unique recovery of small inclusions.