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Convergence of Iterative Laplace Transform Methods for a System of Fractional PDEs and PIDEs Arising in Option Pricing

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Abstract. Iterative Laplace transform methods for fractional partial differential equations and fractional partial integro-differential equations arising in European option pricing with the Lévy α-stable processes and regime-switching or state-dependent jump rates are studied and numerical contour integral methods to inverse the Laplace transform are developed. It is shown that the methods under consideration have the second-order convergence rate in space and spectral-order convergence for Laplace transform inversion.

AMS subject classifications: 35R35, 91G20, 91G60, 91G80

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1. Introduction

Assume that the logarithmic price of the underlying asset $x_t = \log S_t$ follows regime switching model [5–8] and Lévy α-stable processes [3], i.e.

$$\mathrm{d}x_t = (r(\chi(t)) - \gamma(\chi(t)))\,\mathrm{d}t + \sigma(\chi(t))\mathrm{d}L_t^{\alpha,-1}, \quad k = 1,2,\cdots,d, \tag{1.1}$$

where $\chi(t)$ is a continuous-time Markov chain with d-states χ_k and $k \in \mathbb{D} = \{1,2,\cdots,d\}$. Moreover, we also assume that at each state χ_k, the interest rate $r(\chi_k) = r_k$ and the volatility $\sigma(\chi_k) = \sigma_k$ are nonnegative constants, $\gamma(\chi_k) = \gamma_k = -(1/2)\sigma_k^\alpha \sec(\alpha\pi/2)$ is the convexity adjustment and $L_t^{\alpha,-1}$ denotes the maximally skewed log stable process. The stochastic differential equation (1.1) represents a special case of the Lévy α-stable process $L_t^{\alpha,\beta}$ with a tail index $\alpha \in (1,2)$ and the skewed index $\beta = -1$. The detailed financial meaning of the

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model (1.1) are described by Carr *et al.* [3] and Elliott *et al.* [5]. Consider the generator matrix

$$\boldsymbol{Q} = \begin{bmatrix} -q_{11} & q_{12} & q_{13} & & \cdots & q_{1d} \\ q_{21} & -q_{22} & q_{23} & & \cdots & q_{2d} \\ \vdots & \ddots & \ddots & & \ddots & \vdots \\ q_{d-1,1} & q_{d-1,2} & \cdots & & -q_{d-1,d-1} & q_{d-1,d} \\ q_{d,1} & q_{d,2} & \cdots & & q_{d,d-1} & -q_{d,d} \end{bmatrix}$$

of the Markov chain process with constants $q_{kj} \geq 0,\ k, j \in \mathbb{D}$ such that

$$\sum_{j=1, j\neq k}^{d} q_{kj} = q_{kk}, \quad k \in \mathbb{D}.$$

Let $\tau = T - t$ be the time to maturity and $v(k; x, \tau)$ represents the value function of the vanilla European option at the current state $\chi(t) = \chi_k$ and the current log price of asset $x_t = x$. According to the Black-Scholes-Merton model, the option value function $v(k; x, \tau)$ satisfies the following coupled fractional partial integro-differential equations — cf. Refs. [4, 33]:

$$\begin{aligned} \frac{\partial}{\partial \tau} v(k; x, \tau) = {} & \gamma_k \, {}_{-\infty}\mathrm{D}_x^{\alpha} v(k; x, \tau) + (r_k - \gamma_k) \frac{\partial}{\partial x} v(k; x, \tau) \\ & - (r_k + q_{kk}) v(k; x, \tau) + \sum_{j=1, j\neq k}^{d} q_{kj} v(j; x, \tau), \quad k \in \mathbb{D} \end{aligned} \tag{1.2}$$

with the initial condition

$$v(k; x, 0) = (e^x - K)^+ := \max(0, e^x - K), \tag{1.3}$$

and the asymptotic boundary conditions

$$\lim_{x\to-\infty} v(k; x, \tau) = 0, \quad \lim_{x\to+\infty} \left\{ (e^x - K e^{-r_k \tau}) - v(k; x, \tau) \right\} = 0, \tag{1.4}$$

which are similar to the ones proposed by Lee [15] for $k \in \mathbb{D}$. Note that $v(k; x, 0)$ is the payoff function for vanilla European call option.

The Riemann-Liouville fractional derivative ${}_{-\infty}\mathrm{D}_x^{\alpha}$ of $v(k; x, \tau)$ is defined by

$${}_{-\infty}\mathrm{D}_x^{\alpha} v(k; x, \tau) := \frac{1}{\Gamma(n-\alpha)} \frac{\partial^n}{\partial x^n} \int_{-\infty}^{x} (x - y)^{n-\alpha-1} v(k; y, \tau) dy, \quad n - 1 < \alpha < n. \tag{1.5}$$

We recall that two other fractional derivatives — viz. the Grünwald-Letnikov and Caputo derivatives are equivalent to (1.5) if the lower limit in the corresponding integrals is set to $-\infty$, cf. Ref. [25].