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Mixed Spectral Method for Heat Transfer Using Generalised Hermite Functions and Legendre Polynomials

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Abstract. We propose a mixed spectral method for heat transfer in unbounded domains, using generalised Hermite functions and Legendre polynomials. Some basic results on the mixed generalised Hermite-Legendre orthogonal approximation are established, which plays important roles in spectral methods for various problems defined on unbounded domains. As an example, the mixed generalised Hermite-Legendre spectral scheme is constructed for anisotropic heat transfer. Its convergence is proven, and some numerical results demonstrate the spectral accuracy of this approach.

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Key words: The scaled generalised Hermite functions, mixed spectral method, anisotropic heat transfer, unbounded domains.

1. Introduction

Spectral methods have developed rapidly over the past three decades. The main merit is high accuracy, but earlier spectral methods were only applicable to bounded domains — cf. [1,3–9] and references therein. However, increasingly attention turned to problems on unbounded domains, where direct approaches have been based upon certain orthogonal approximations — e.g. the Hermite and Laguerre spectral method now commonly used for differential equations on unbounded domains [1,10–20,22–30,32–34]. Hermite polynomials have previously been used, but for some applications the solutions decay to zero at infinity — e.g. in heat transfer problems or when the differential equations involve variable coefficients. Since the weight function e^{-x^2} (or e^{x^2}) destroys crucial the symmetry and positive definiteness of bilinear operators [1,12–14,19], in such cases it is more appropriate to exploit Hermite functions with weight $\chi(x) \equiv 1$ — e.g. in the recent numerical simulation

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of the Dirac equation [16]. However, Tang [27] pointed out that standard Hermite functions sometimes work poorly in practice because too many Hermite functions are required, and consequently introduced the scaled generalised Hermite functions

$$H_n^\alpha(x) = \frac{1}{\sqrt{2^n n!}} H_n(\alpha x) e^{-\frac{1}{2}\alpha^2 x^2}, \quad \alpha > 0\,,$$

as basis functions that are mutually orthogonal with weight $\chi(x) \equiv 1$. Xiang & Wang [34] considered the spectral method for the generalised Ginzburg-Landau equation, using these scaled generalised Hermite functions. Tang [27] also considered how to choose a proper scaling factor for a class of functions that decay at infinity at least like $e^{-\gamma x^2}$ (with $\gamma > 0$).

We consider anisotropic heat transfer in an infinite plate, where the domain is $\Omega = \Lambda \times I$ where $\Lambda = \{x \mid -\infty < x < \infty\}$ and $I = \{y \mid |y| < 1\}$ and the boundary is denoted by $\Gamma = \{(x,y) \mid -\infty < x < \infty,\ |y| = 1\}$. For brevity, we write $\partial_z W = \partial W / \partial z$, etc. Then the anisotropic heat transfer in Ω is governed by

$$\begin{cases} \partial_t W(x,y,t) + a\partial_x W(x,y,t) + b\partial_y W(x,y,t) & \\ \quad -\nu\partial_x^2 W(x,y,t) - \mu\partial_y^2 W(x,y,t) = F(x,y,t)\,, & (x,y) \in \Omega\,,\ 0 < t \le T\,, \\ W(x,y,t) = W_1(x,y,t)\,, & (x,y) \text{ on } \Gamma\,,\ 0 < t \le T\,, \\ W(x,y,0) = W_0(x,y)\,, & (x,y) \in \bar{\Omega}\,, \end{cases} \tag{1.1}$$

where $W(x,y,t)$ denotes the temperature, positive constants ν and μ are the conductivities in the respective directions x and y, the parameters a and b are convective constants, and the specified functions $F(x,y,t)$ and $W_0(x,y)$ describe the heat source and the initial state, respectively. Guo & Wang [19] used Hermite functions as the orthogonal basis functions (associated with the weight function e^{x^2}), and since $W(x,y,t)$ decays very fast as $|x| \to \infty$ it is proved better to adopt the generalised Hermite functions [13, 16, 19]. However, it is notable that problem (1.1) is not well posed in the weighted Sobolev space. Our aim here is to develop a mixed generalised Hermite-Legendre approximation with application to the numerical solution of problem (1.1) using the above scaled generalised Hermite functions $H_n^\alpha(x)$ and Legendre polynomials, associated with simple alternative Galerkin variational formulations. Since such a formulation can approximate the solution directly and be implemented easily, invoking the scaled generalised Hermite functions leads to a much simplified analysis and more precise error estimates [34]. Moreover, the adjustable parameter α offers great flexibility to match the asymptotic behaviour of the exact solutions at infinity [31].

In Section 2, we establish some results on the mixed generalised Hermite-Legendre approximation, which play important roles in the numerical analysis of spectral methods for various differential equations in unbounded domains. In Section 3, we construct our mixed generalised Hermite-Legendre spectral scheme for the problem (1.1) and prove its convergence. Some numerical results indicating the high accuracy of proposed algorithm in Section 4, and our concluding remarks are in Section 5.