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Recursive Identification of Wiener-Hammerstein Systems with Nonparametric Nonlinearity

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Abstract. A recursive scheme is proposed for identifying a single input single output (SISO) Wiener-Hammerstein system, which consists of two linear dynamic subsystems and a sandwiched nonparametric static nonlinearity. The first linear block is assumed to be a finite impulse response (FIR) filter and the second an infinite impulse response (IIR) filter. By letting the input be a sequence of mutually independent Gaussian random variables, the recursive estimates for coefficients of the two linear blocks and the value of the static nonlinear function at any fixed given point are proven to converge to the true values, with probability one as the data size tends to infinity. The static nonlinearity is identified in a nonparametric way and no structural information is directly used. A numerical example is presented that illustrates the theoretical results.

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Key words: Wiener-Hammerstein system, nonparametric nonlinearity, recursive estimate, strong consistence.

1. Introduction

The Wiener-Hammerstein (W-H) system comprises two linear dynamic subsystems with a sandwiched static nonlinearity — cf. Fig. 1. A system consisting of the first two blocks is called a Wiener system, whereas a system consisting of the last two blocks is known as a Hammerstein system. Thus W-H systems are a natural extension of Wiener systems and Hammerstein systems, which are all important for modelling many real phenomena. Applications include a distillation column [1], a pH control process [2] for Wiener systems, a cat visual cortex for Hammerstein systems [3], and a light flickering severity-meter for W-H systems [4]. Identification of these systems has therefore been an active research

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area for many years — e.g. see Refs. [5–12] for Wiener systems, [13–16] for Hammerstein systems, and [4, 17–21] for W-H systems.

Since a W-H system is a combination of a Wiener system and a Hammerstein system, a natural starting point is to identify those two simpler components before considering a W-H system. The identification for Wiener systems is much more difficult than for Hammerstein systems, with the essential difference in identification of these two types of nonlinear systems roughly as follows. In the case of a Hammerstein system, if a sequence of independent identically distributed (i.i.d.) random variables is selected as input, it still remains a sequence of i.i.d. random variables after passing through the static nonlinearity of the Hammerstein system. Hence identifying the linear subsystem in a Hammerstein system is a standard Auto-Regressive and Moving Average (ARMA) model issue, where the random variables (outputs of the nonlinearity) in the Moving Average (MA) part may not be zero-mean. However, it is quite different in the case of a Wiener system for i.i.d. input. The intermediate signal (the output of the linear subsystem) is no longer mutually independent, and in general has a complicated distribution unless the input of the linear subsystem is Gaussian. Moreover, the unboundedness of any Gaussian signal may cause additional difficulties in the convergence analysis when using a Gaussian input. This explains why more restrictive conditions are used and less theoretical results are obtained for identifying Wiener systems in comparison with Hammerstein systems. Thus the chief key difficulty in identification for W-H systems is how to identify Wiener systems.

Due to the weak properties of the intermediate signal of a Wiener system, it is no wonder that the static nonlinearity is usually assumed to be invertible or described in a simple parametric form (typically as a low order polynomial). If the order of the polynomial is high, the number of whole terms in the system input-output expansion will be huge. However, even under such strict restrictions, it is still quite difficult to establish convergence results.

Nonparametrisation for the nonlinearity is another way, and some weak convergence results have been established in this context [5]. Several recursive identification algorithms for Wiener systems have also been proposed under different system settings [7–9], and their strong consistences are justified. The first guaranteed consistent recursive identification algorithm for the Wiener system is given in Ref. [7], where it is unnecessary for the nonparametric nonlinear function to be inversive. However, a certain amount of data is excluded for usage there due to a technical reason, and this drawback was cured in Ref. [9]. The Wiener system with a more general linear subsystem is introduced and considered in Ref. [8]. The main idea there is that the coefficients of the linear subsystem are estimated by a system of equations comprising cross-correlation coefficients of input and output, when the nonlinearity is estimated by the estimated intermediate signal and the output with the kernel function technique. It seems possible that nonparametrisation may be used in identifying W-H systems.

Although a number of identification algorithms have been proposed for W-H system (e.g. see [4, 17, 19–21]), relevant theoretical guarantees for the consistency of the corresponding identification algorithms seem rare. In Ref. [17], the best linear approximation method carried out to serve as an initial estimation for W-H systems is actually a para-