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Lump and Rogue Wave Solutions of a Reduced $(3+1)$-Dimensional Shallow Water Equation

Jiayue Gu and Huanhe Dong*

College of Mathematics and Systems Science, Shandong University of Science and Technology, Qingdao, 266590, China.

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Abstract. Considering a reduced $(3+1)$-dimensional shallow water equation, we use Hirota formulation and symbolic calculation to derive positive lump solitons rationally localised in all directions of the (x, y)-plane. The interaction of the lump and one stripe solitons is studied. Numerical experiments show that the collision of such solutions is completely inelastic and the lump soliton is swallowed by the stripe one. Exploring the interaction of the lump and a couple of resonance stripe solitons, we note that the lump soliton transforms into a ghost soliton. Most of the time it remains hidden in stripe solitons, but appears at a certain time and fades after that.

AMS subject classifications: 35Q51, 35Q53, 37K40

Key words: Lump solution, rogue wave, (3+1)-dimensional shallow water equation, Hirota bilinear operator.

1. Introduction

Solitons form a special class of solutions of nonlinear equations and are related to many significant properties of non-linear systems such as infinite conservation laws [7,11,12,28], Hamiltonian structures [13,27,33,37], Lax pairs [4,5,29], bilinear formalism [6,9,10,16] and Painlevé property [14]. An important part of the corresponding studies comprises the integrability of nonlinear evolution equation (NLEEs). The methods used to derive exact solutions of such equations include classic inverse scattering [1], Bäcklund transformation [21,22,31,41], ND trial functions [15,34], variable separation [23,24] and bilinear method. The last one turned out to be very efficient and was used to obtaine multi-soliton solutions [2], rogue wave solutions [38], Pfaffian solutions [3]. It is worth noting that some types of rational solutions attracted a considerable attention recently. Thus rogue waves have been studied by various methods in [30,32,38], and exact lump solutions are derived for the KPI [17], BKP [35], p-gKP and p-gBKP [18], $(3+1)$-dimensional shallow water wave [39] equations and so on. The lump and interaction solutions obtained from the quadratic

*Corresponding author. *Email address:* `mathsdong@126.com` (H. Dong)

function method are considered in [19, 20, 36, 40, 42]. Certain solitons interact with each other when they collide. There are two types of collision — viz. elastic and inelastic. Thus after the collision with soliton solutions, the lump solution preserves the shape, amplitude and velocity — i.e. the collision is completely elastic [8]. On the other hand, the lump solution is swallowed by stripe solutions [25], so that this collision is completely inelastic.

In this work we focus on a $(3+1)$-dimensional shallow water equation [26, 39]—viz.

$$u_{xxxy}+3u_yu_{xx}+3u_xu_{xy}+2u_{yt}-3u_{xz}=0. \tag{1.1}$$

It has been studied by different methods, including Grammian and Pfaffian solutions [26] and the rational function method [39]. Nevertheless, finding exact rogue wave solutions of the Eq. (1.1) still remains an important task in many applications. In Section 2, we derive lump solution with the positive quadratic function for a reduced (3+1)-dimensional shallow water equation. Section 3 describes the interaction between lump and one stripe solitons. In Section 4, we extend the positive quadratic function to a combination of hyperbolic cosine functions, discuss the interaction of lump and a pair of resonance stripe solitons and describe their kinestate.

2. Lump Solutions of a Shallow Water Equation

In this section, we study the lump solution to $(3+1)$-dimensional shallow water equation with positive quadratic function. Bilinear formulism of a $(3+1)$-dimensional shallow water equation is

$$(D_x^3D_y-D_yD_t-D_xD_z)f\cdot f=0, \tag{2.1}$$

where D is the Hirota bilinear operator defined by

$$D_t^mD_x^nF(t,x)\cdot G(t,x)=\frac{\partial^m}{\partial s^m}\frac{\partial^n}{\partial y^n}F(t+s,x+y)G(t-s,x-y)\,|_{s=0,y=0},$$

for $m,n=0,1,2,\cdots$.

We only consider a one type of lump solutions of the Eq. (2.1). If spatial variable $z=x$, one can use the transformation $u=2(\ln f)_x$ and rewrite the Eq. (2.1) as

$$f_{xxxy}f-f_{xxx}f_y-3f_{xxy}f_x+3f_{xx}f_{xy}-f_{yt}f+f_yf_t-f_{xx}f+f_x^2=0. \tag{2.2}$$

Let us assume that

$$f=g^2+h^2+a_9,\quad g=a_1x+a_2y+a_3t+a_4,\quad h=a_5x+a_6y+a_7t+a_8, \tag{2.3}$$

where constants $a_i, 1\le i\le 9$ will be determined later. Substitution of f into the Eq. (2.2) and symbolic calculations lead to the relations

$$\begin{aligned}&a_3=\frac{a_2a_5^2-2a_5a_1a_6-a_2a_1^2}{a_6^2+a_2^2},\quad a_7=\frac{a_6a_1^2-a_6a_5^2-2a_2a_5a_1}{a_6^2+a_2^2},\\&a_9=\frac{3(a_6^2+a_2^2)(a_1^3a_2+a_1^2a_5a_6+a_1a_2a_5^2+a_5^3a_6)}{a_1^2a_6^2-2a_1a_2a_5a_6+a_2^2a_5^2}.\end{aligned} \tag{2.4}$$