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Delay Induced Hopf Bifurcation in a Nonlinear Innovation Diffusion Model: External Influences Effect

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Abstract. A nonlinear innovation diffusion model which incorporates the evaluation stage (time delay) is proposed to describe the dynamics of three population classes for non-adopter and adopter densities. The local stability of the various equilibrium points is investigated. It is observed that the system is locally asymptotically stable for a delay limit and produces periodic orbits via a Hopf bifurcation when evaluation period crosses a critical value. Applying normal form theory and center manifold theorem, we study the properties of the bifurcating periodic solutions. The model shows that the adopter population density achieves its maturity stage faster if the cumulative density of external influences increases. Several numerical examples confirm our theoretical results.

AMS subject classifications: 34C23, 34D20, 37L10, 92D25

Key words: Evaluation period, stability analysis, Hopf bifurcation, normal form theory, center manifold theorem.

1. Introduction

Newly appearing products can be designated into three basic groups: new brands, inventions and retrofit products. Most of the successful new products are rather innovative than inventive. Therefore, the diffusion strategies for new products are mainly focused on innovation procedures [22]. According to Rogers [42], a technological innovation is a design for instrumental action that reduces the uncertainty in the cause-effect relationships involved in achieving the desired outcome. Rogers also concludes that the diffusion of innovations consists in five steps — viz. exposure to innovations, interest in more information,

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evaluation of innovations in specific situations, trials and adoption of innovations. This five step innovation diffusion process can be rearranged into two-step flow process when the media promote the opinion of the adopter leaders, which, in turn, influences other people to adopt the product —cf. Refs. [5–7, 21, 38, 42].

The papers [2, 15, 39] deal with growth models for consumer durables and discuss the diffusion pattern under external and internal influences such as advertisements and oral communications. The Bass model played an important role in understanding the spread of new products and the phenomena responsible for these processes [8, 9, 18, 35–37, 45, 46, 49, 50]. It has been widely used in market analysis and demand forecasting in different situations [17, 24, 41]. The diffusion process is often modelled via a two-stage single differential equation approach, representing the epidemics manner in which the penetration and adoption of the innovation are simultaneously influenced by external and internal sources [2,30,40,55]. The price and advertising variables are incorporated in these models to determine the basic parameters of the corresponding differential equation [21, 23, 44]. A one product innovation diffusion model in patch environment is considered in [55], and the competitive innovation diffusion model for three products is discussed in [53, 57, 59]. Further, the innovation diffusion model has been extended to n competing products and the system is proved to be globally stable by constructing a Lyapunov function [58]. A multi-generation product model is proposed and an empirical study is taken on the telecommunication products. It proves that the multi-generation products diffusion model is better than other models in terms of forecast accuracy [60]. Various models with time delay to exhibit product evaluation stages have been proposed [12, 14, 26, 27, 54] and distinct delayed models incorporating the evaluation stage of a product predicting the influence of word-of-mouth communications have been presented [1, 3, 10, 35, 54, 56].

Centrone [4] investigated a binomial innovation diffusion model for a variable size market for demographic processes. Shukla *et al.* [48] assumed that such processes are affected by variable external factors, intrinsic growth rate, emigration or death rate etc. The main effect of external forces is to reach the equilibrium of adopters at a much faster rate [48]. Considering a time delay model simulating the adoption stages, Fanelli *et al.* [12] underscored the importance of external factors such as government policy and production cost.

In this paper, we consider a nonlinear innovation diffusion model, which includes demographic processes along with an evaluation period. Our goal is to analyse the Hopf bifurcation and to study the stability of bifurcating periodic solutions. We generalise the models [4, 48] by introducing logically growing non-adopter classes of immature and mature populations and other external and internal factors. The external factors for the adoption of an innovation include media influence, while internal are related to interpersonal communication between adopters and potential adopters. The evaluation and decision-making stages reflect a time delay in the system. The innovation diffusion patterns connected with the stability are qualitative in nature, and the Hopf bifurcation theory is used to classify diffusion patterns in design and in future trend forecasting.

The paper is organised as follows. In Section 2, we introduce an innovation diffusion model based on delay differential equations. Section 3 is concerned with the positivity