East Asian Journal on Applied Mathematics
doi: 10.4208/eajam.030915.210416a

Vol. **6**, No. 3, pp. 290-313
August 2016

Direct Gravitational Search Algorithm for Global Optimisation Problems

Ahmed F. Ali[1,2] and Mohamed A. Tawhid[2,3,*]

[1] *Department of Computer Science, Faculty of Computers & Informatics, Suez Canal University, Ismailia, Egypt.*
[2] *Department of Mathematics and Statistics, Faculty of Science, Thompson Rivers University, Kamloops, BC, Canada V2C 0C8.*
[3] *Department of Mathematics and Computer Science, Faculty of Science, Alexandria University, Moharam Bey 21511, Alexandria, Egypt.*

Received 3 September 2015; Accepted (in revised version) 21 April 2016.

Abstract. A gravitational search algorithm (GSA) is a meta-heuristic development that is modelled on the Newtonian law of gravity and mass interaction. Here we propose a new hybrid algorithm called the Direct Gravitational Search Algorithm (DGSA), which combines a GSA that can perform a wide exploration and deep exploitation with the Nelder-Mead method, as a promising direct method capable of an intensification search. The main drawback of a meta-heuristic algorithm is slow convergence, but in our DGSA the standard GSA is run for a number of iterations before the best solution obtained is passed to the Nelder-Mead method to refine it and avoid running iterations that provide negligible further improvement. We test the DGSA on 7 benchmark integer functions and 10 benchmark minimax functions to compare the performance against 9 other algorithms, and the numerical results show the optimal or near optimal solution is obtained faster.

AMS subject classifications: 49K35, 90C10, 68U20, 68W05

Key words: Gravitational search algorithm, direct search methods, Nelder-Mead method, integer programming problems, minimax problems.

1. Introduction

A gravitational search algorithm (GSA) is a population based meta-heuristic algorithm developed by Rashedi [32]. This and many other such meta-heuristic algorithms (including ant colony optimisation (ACO) [7], artificial bee colony [16], particle swarm optimisation (PSO) [17], bacterial foraging [28], bat algorithm [42], bee colony optimisation (BCO) [36], wolf search [35], cat swarm [6], firefly algorithm [41], and fish swarm/school

*Corresponding author. *Email addresses:* `ahmed_fouad@ci.suez.edu.eg` (A. F. Ali), `Mtawhid@tru.ca` (M. A. Tawhid)

 ©2016 Global-Science Press

[20]) have been used to solve unconstrained and constrained optimisation problems and their applications, and sometimes to minimax and integer programming problems.

Integer programming and combinatorial optimisation problems optimise functions of many variables subject to some problem specific constraints and integrality restrictions imposed on all or some of the variables, with many real applications such as airline crew scheduling, production planning, Internet routing, VLSI (very large scale integration), and packing and cutting. A combinatorial optimisation problem can often be modelled as an integer program [25, 26], but nevertheless these problems can be very difficult to solve as they are NP-hard [10]. Given their difficulty but practical importance, a large number of solution techniques for NP-hard integer and combinatorial optimisation problems have been proposed.

The available algorithms can be classified into two main classes — viz. exact and approximate algorithms. One of the most famous exact integer programming algorithms is Branch and Bound (BB), but suffers from high complexity in solving large scale problems as it explores hundreds of nodes in big tree structures. Recently, approximate algorithms such as swarm intelligence algorithms (ant colony algorithm [14, 15], artificial bee colony algorithm [1, 37], particle swarm optimisation algorithm [29], cuckoo search algorithm [38] and firefly algorithm [3]) have been used to solve integer programming problems

Minimax problems appear in many engineering areas such as optimal control, engineering design, discrete optimisation, Chebyshev approximation, game theory, computer-aided design, and circuit design — e.g. see Ref. [8, 43]. Moreover, any nonlinear programming problem with nonlinear constraints can be transformed into an equivalent unconstrained minimax problem [2]. There are some algorithms to solve minimax problems, involving the solution of a sequence of smoothing problems that approximate the minimax problems in the limit [21, 30, 40]. Thus these algorithms generate a sequence of approximations, which converges to the Kuhn-Tucker point of the given minimax problem for a decreasing sequence of positive smoothing parameters. However, a drawback is that these parameters can become small too quickly such that the smooth problems become significantly ill-conditioned. Some swarm intelligence algorithms have been applied to solve minimax problems (e.g. the PSO [29]), but their main drawback in solving minimax and integer programming problems is that they are computationally expensive.

We propose a new hybrid gravitational search algorithm and Nelder-Mead method called the Direct Gravitational Search Algorithm (DGSA), to overcome the slow convergence of the standard gravitational search algorithm (GSA) for solving global optimisation problems. The Nelder-Mead direct search method can accelerate the search and improve the convergence, avoiding additional iterations that provide negligible further improvement. The integer programming, minimax problems and the Nelder-Mead method are presented in Section 2. Section 3 summarises the main concepts of a gravitational search algorithm (GSA). The main structure of the proposed DGSA is presented in Section 4, and then tested on 7 integer programming and 10 minimax benchmark problems. The experimental results in Section 5 show that the DSGA can obtain the optimal or near optimal solution in most cases, and our concluding remarks are in Section 6.