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New Parallel Algorithm for Convection-Dominated Diffusion Equation

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Abstract. A new parallel algorithm for convection-dominated diffusion equation is proposed. To derive an approximate solution of the problem at the same time level, we use two domain decomposition methods. The average of the solutions obtained is set to be new numerical solution. The algorithm demonstrates a high accuracy and is suitable for parallel computing. Approximate solution of two-dimensional convection-dominated diffusion equations is also considered. Numerical examples show the efficiency and reliability of the algorithm.

AMS subject classifications: 65M06, 65M12, 65Y05

Key words: Convection-dominated diffusion equation, finite difference, unconditional stability, parallelism.

1. Introduction

Parallel algorithms are a relatively new development in high performance computing, and the parallel difference method for parabolic equations attracted a considerable interest. In 1983, Evans and Abdullah [1] used the Group Explicit method in Saul'yev asymmetric schemes [2] for parabolic equations. Two years later, Evans [3] constructed an Alternating Group Explicit (AGE) scheme for diffusion equations. An Alternating Segment Explicit-Implicit (ASE-I) method and Alternating Segment Crank-Nicolson (ASC-N) scheme are constructed in [4–7]. Afterwards, alternating segment algorithms (AGE scheme, ASE-I scheme and ASC-N scheme) became popular and efficient methods for parabolic equations, including convection-diffusion equation [8–10], dispersive equation [11–15], forth-order parabolic equation [16, 17], nonlinear third-order KdV equation [18] and nonlinear Leland equation [19]. On the other hand, domain decomposition method (DDM) for partial differential equations has been developed in Refs. [20–33]. The concept called "intrinsic parallelism" was presented in [34–36]. In 1999, alternating difference schemes with intrinsic parallelism for two-dimensional parabolic systems were introduced in [37], and the

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unconditional stability analysis was given. The unconditionally stable domain decomposition method was obtained by the alternating technique in [38–40]. In fact, the alternating segment algorithm is also a domain decomposition method, suitable for parallel computation and unconditionally stable. On the other hand, the accuracy of this algorithm is not satisfactory.

The new parallel process for the convection-dominated diffusion equation, developed in this work, is inspired by the alternating segment algorithm. Considering the one-dimensional convection-dominated diffusion equation as a model, we describe the main principles of parallel processes. The new parallel algorithm consists of two DDMs, each of which is used in computations at $(n+1)$-st time level utilising the numerical solution at the time level n. It can be described as follows:

1. DDM I is employed to find the solution V^{n+1} at the time level $(n+1)$ from known approximate solutions U^n at the time level n.

2. DDM II is employed to find the solution W^{n+1} at the time level $(n+1)$ from known approximate solutions U^n at the time level n.

3. The values $U^{n+1} := (V^{n+1} + W^{n+1})/2$ are set as numerical solutions at the time level $(n+1)$.

In Section 2, we introduce a modified Crank-Nicolson scheme and Saul'yev asymmetric difference schemes, and describe a parallel algorithm for one-dimensional convection-dominated diffusion equation. The error analysis and unconditional stability of this algorithm are discussed in Sections 3. In Section 4, the algorithm is used in ADI method for two-dimensional convection-dominated diffusion equations. Numerical examples presented in Section 5 demonstrate the accuracy and efficiency of this algorithm.

2. Algorithm Description

We consider the one-dimensional convection-dominated diffusion equation

$$\begin{aligned}
&\frac{\partial u}{\partial t} - a\frac{\partial^2 u}{\partial x^2} + b\frac{\partial u}{\partial x} = f(x,t), \quad x \in (0,l), \ t \in (0,T], \\
&u(x,0) = u_0(x), \quad x \in [0,l], \\
&u(0,t) = g_0(t), \ u(1,t) = g_1(t), \quad t \in (0,T],
\end{aligned} \tag{2.1}$$

where $|b| \gg a$ for a given positive constant a.

Let $h = l/m$ and $\tau = T/N$ be, respectively, the spatial and temporal step sizes, and $x_j = jh, \ j = 0, \ 1, \ \cdots, \ m$ and $t_n = n\tau, \ n = 0, \ 1, \ \cdots, \ N$ be the corresponding meshes. If u_j^n refers to an approximate solution at the the point (x_j, t_n), then one can consider the