Abundant Mixed Lump-Soliton Solutions to the BKP Equation

Jin-Yun Yang, Wen-Xiu Ma, and Zhenyun Qin

Abstract. Applying Maple symbolic computations, we derive eight sets of mixed lump-soliton solutions to the $(2 + 1)$-dimensional BKP equation. The solutions are analytic and allow the separation of lumps and line solitons.

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1. Introduction

It is well known that solitons describe various significant nonlinear phenomena in nature [1, 39] and the Hirota bilinear method provides a powerful tool for solving integrable equations [16]. Positons and complexitons are other typical solutions of integrable equations [25, 43], and the interaction between different classes of solutions leads to a better understanding of nonlinear phenomena [35]. In particular, the long wave limits of solitons generate lump solutions, rationally localized solutions in all directions in space, and many
other ones — cf. Refs. [1,40]. Hirota bilinear forms play a crucial role in finding such exact solutions but the algorithm heavily relies on try and error experiments [5,16].

Let us recall [30] that the KP equation

\[(u_t + 6uu_x + u_{xxx})_x - u_{yy} = 0,\]

has the following set of lump solutions:

\[
u = 2(\ln f)_{xx}, \quad f = (a_1 x + a_2 y + \frac{a_1 a_2^2 - a_1 a_6^2 + 2 a_2 a_5 a_0}{a_1^2 + a_5^2} t + a_4)^2
\]
\[+ (a_5 x + a_6 y + \frac{2 a_1 a_2 a_6 - a_2^2 a_5 + a_5 a_6^2}{a_1^2 + a_5^2} t + a_8)^2 + \frac{3(a_1^2 + a_5^2)^3}{(a_1 a_6 - a_2 a_5)^2},\]

where \(a_i\) are arbitrary parameters such that \(a_1 a_6 - a_2 a_5 \neq 0\). This set contains a subset of lump solutions of the form

\[u = 4 - \frac{(x + ay + (a^2 - b^2)t)^2 + b^2(y + 2at)^2 + 3/b^2}{((x + ay + (a^2 - b^2)t)^2 + b^2(y + 2at)^2 + 3/b^2)^2},\]  

(1.1)

with two free parameters \(a\) and \(b\) [38]. The situation is not unique and there are many integrable equations with lump solutions — e.g. three-dimensional three-wave resonant interactions [21], the BKP equation [14,45], the Davey-Stewartson equation II [40], the Ishimori-I equation [17] and so on. Besides, non-integrable equations, such as \((2 + 1)\)-dimensional generalized KP, BKP and Sawada-Kotera equations, also have lump solutions — cf. Refs. [8,32,36,48,53].

It is worth noting that the general rational solutions of integrable equations have been derived within the framework of Wronskian, Casoratian, Grammian and Pfaffian formulations [1,16]. The set of equations studied contains a variety of physically significant equations such as the KdV and Boussinesq equations, the nonlinear Schrödinger equation in \((1 + 1)\)-dimensions, the KP and BKP equations in \((2 + 1)\)-dimensions, and the Toda lattice equation in \((0 + 1)\)-dimensions [2,7,13,34,35]. General rational solutions of nonlinear partial differential equations — e.g. generalized bilinear differential equations have been also discussed [3,37,47,50–52].

Here we consider a \((2 + 1)\)-dimensional BKP equation of [9,18] — viz.

\[P_{BKP}(u) := (u_t + 15uu_{xxx} + 15u_x^3 - 15u_x u_y + u_{5x})_x - 5u_{xxx} - 5u_{yy} = 0.\]  

(1.2)

This is a first member in the BKP integrable hierarchy [6,41], represented by the \((2 + 1)\)-dimensional generalization of the Caudrey-Dodd-Gibbon-Sawada-Kotera equation

\[v_t + 15 vv_{xxx} + 15v_x v_{xx} + 45v^2 v_x + v_{5x} = 0.\]  

(1.3)

If \(v = u_x\) and the function \(u\) depends on \(x\) and \(t\) only, then (1.2) becomes the Eq. (1.3).

The underlying spectral problem

\[-\phi_{yy} + \phi_{xxx} + (3\nu - \lambda)\phi = 0,\]  

(1.4)