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Optimal Strategy for Limit Order Book Submissions in High Frequency Trading

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Abstract. An optimal selection problem for bid and ask quotes subject to a stock inventory constraint is investigated, formulated as a constrained utility maximisation problem over a finite time horizon. The arrivals of buy and sell orders are governed by Poisson processes, and a diffusion approximation is employed on assuming the Poisson arrivals intensity is sufficiently large. Using the dynamic programming principle, we adopt an efficient numerical procedure to solve this constrained utility maximisation problem based on a successive approximation algorithm, and conduct numerical experiments to analyse the impacts of the inventory constraint on a dealer's terminal profit and stock inventory level. It is found that the stock inventory constraint significantly affects the terminal stock inventory level.

AMS subject classifications: 60K25, 68M20, 91A80

Key words: High-frequency trading, Limit Order Book (LOB), Diffusion Approximation, Hamilton-Jacobi-Bellman (HJB) Equation.

1. Introduction

High-frequency trading uses complex algorithms to analyse multiple markets and execute orders rapidly. Unlike in regular trading, an investment position in high-frequency trading may be held from fractions of a second to several hours, and it is characterised by

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short portfolio holding periods. There has been a rapid growth in high-frequency trading since the U.S. Securities and Exchange Commission (SEC) authorised electronic exchanges in 1998. Thus in the early 2000s high-frequency trading accounted for fewer than 10% of equity orders, but by 2010 an estimated 73% of exchange volume came from high-frequency trading orders in the United States. In Europe, high-frequency trading accounts for about 40% of equity orders volume, and in Asia about 5% to 10% with potential for rapid growth [6]. One high-frequency trading strategy involves placing a limit order to sell (or ask) or a buy limit order (or bid) to earn the bid-ask spread. To maximise the terminal profit, the dealer faces an inventory risk arising from uncertainty in the stock's price and a transactions risk due to Poisson arrivals of market buy and sell orders. To consider these two sources of risk, Ho & Stoll [8] developed a model to analyse the optimal prices for a monopolistic dealer in a single stock. Their results indicate that the optimal bid and ask quotes are around the "true" price of the stock, and Ho & Stoll [8] also pointed out that the bid and ask quotes are related to the reservation prices of the dealers when they are in competition. Based on these results, Avellaneda & Stoikov [1] studied optimal submission strategies by assuming the "true" price of the stock is modelled as a Brownian motion.

High-frequency traders moving in and out of short-term positions at high volumes aim to capture sometimes only a fraction of a cent in profit on every trade. High-frequency trading firms do not consume significant amounts of capital, accumulate positions or hold their portfolios overnight [11]. In a high volatility market, the risk of holding an overnight position is relatively high, and in any case the payment of interest from margin for an overnight position reduces the profit of high-frequency trading. Thus in practice high-frequency traders may not hold their portfolios overnight, corresponding to a stock inventory constraint on the optimal selection problem for bid and ask quotes.

Song *et al.* [16] employed a diffusion approximation to Poisson arrivals of market orders, such that the stock inventory level and the wealth dynamics can be approximated by Wiener processes. For practical applications in various fields, normal distributions often provide a reasonable approximation to a variety of data. Feller [3] pointed out that the Poisson distribution $Poi(\lambda)$ can be well approximated by the normal distribution $N(\lambda, \lambda)$, when the intensity is large enough. In this article, a diffusion approximation to Poisson arrivals of the market buy and sell orders is adopted.

There are many applications of a diffusion approximation. For example, Kobayashi [10] pointed out the queueing processes of various service stations that interact with each other can be approximated by a vector-valued Wiener process. Nagaev *et al.* [13] assumed that stock price evolution is described by a Markov chain, and obtained a simple but powerful approximation formula for the evolution by applying a diffusion approximation to the Markov chain. In insurance risk theory, a diffusion approximation has been used to approximate insurance surplus processes described by compound Poisson processes — e.g. see Grandell [4], where as in Song et al. [16] the dynamic programming principle is used to derive an Hamilton-Jacobi-Bellman (HJB) equation. The solution of the optimal submission problem of bid and ask quotes can then be obtained by solving the HJB equation; and we employ the successive approximation algorithm introduced by Chang & Krishna [2] to solve this second-order partial differential equation (PDE), coupled with an optimisation