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Spherical Geometry HOC Scheme to Capture Low Pressures within a Wake

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Abstract. The solution of the pressure Poisson equation in spherical polar coordinates using a higher order compact (HOC) scheme effectively captures low pressure values in the wake region for viscous flow past a sphere. In the absence of an exact solution, the fourth-order of accuracy of the results is illustrated. Low pressure circular contours occur in the wake region when the Reynolds number $Re = 161$, which is a lower value than previously identified in the literature, and closed pressure contours appear in two regions when $Re = 250$.

AMS subject classifications: 76D05, 35Q35, 65N06

Key words: Pressure fields, low pressure, spherical geometry, higher order compact scheme, Navier-Stokes equations.

1. Introduction

Higher order compact (HOC) schemes have been used elsewhere to describe flow quite accurately — e.g. at weather warning centres and in global ocean modelling. Simpler second-order spatially accurate schemes have also been used in many computational fluid dynamics problems, but they may fail or require smaller than acceptable grid step sizes to capture the flow phenomena when the domain is large. The pressure in the entire computational domain is often important in describing the flow behaviour. Calculations by Fornberg [1] and Johnson & Patel [2] for steady viscous flow past a sphere in the entire computational domain are at most second order accurate. Recently, Sekhar *et al.* [3] developed a higher order compact scheme to solve the Navier-Stokes equations in spherical polar co-ordinates for viscous flow past a sphere. This article extends that scheme to solve the corresponding pressure Poisson equation in spherical polar coordinates, and the low pressure behaviour predicted within the wake region differs from some of the earlier results in Refs. [1,2] and references therein.

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2. Basic Equations

We consider steady axisymmetric viscous incompressible flow past a sphere placed in a uniform stream with velocity U_∞ from left to right. The governing continuity and momentum equations are

$$\nabla\cdot \boldsymbol{v} = 0\,, \tag{2.1}$$

$$\boldsymbol{v}\cdot\nabla\boldsymbol{v} = -\nabla p + \frac{2}{Re}\nabla^2\boldsymbol{v}\,, \tag{2.2}$$

where the relevant length scale is the radius of the sphere a but we have chosen to refer to the diameter of the sphere (and the kinematic viscosity coefficient ν) in defining the Reynolds number

$$Re = \frac{2\,U_\infty\, a}{\nu}\,.$$

The dimensionless radius of the sphere is thus $r = 1$, while the dimensionless velocity $\boldsymbol{v}$ corresponds to dividing the dimensional velocity by the uniform stream velocity U_∞. In terms of a stream function ψ, in spherical polar coordinates the respective dimensionless radial and transverse velocity components are

$$v_r = \frac{1}{r^2\sin\theta}\frac{\partial\psi}{\partial\theta}\,, \qquad v_\theta = -\frac{1}{r\sin\theta}\frac{\partial\psi}{\partial r}\,, \tag{2.3}$$

such that the continuity equation (2.1) is immediately satisfied. From the well known vector identities

$$\boldsymbol{v}\cdot\nabla\boldsymbol{v} = \nabla\left(\frac{1}{2}v^2\right) - \boldsymbol{v}\times(\nabla\times\boldsymbol{v})$$

and

$$\nabla\times\nabla\times\boldsymbol{v} = \nabla(\nabla\cdot\boldsymbol{v}) - \nabla^2\boldsymbol{v}\,,$$

the momentum equation (2.2) may be rewritten

$$\nabla\left(\frac{1}{2}v^2\right) - \boldsymbol{v}\times\boldsymbol{\omega} = -\nabla p - \frac{2}{Re}\nabla\times\boldsymbol{\omega} \tag{2.4}$$

where $\boldsymbol{\omega} = \nabla\times\boldsymbol{v}$ is the vorticity, and then taking the curl we obtain

$$\nabla\times(\boldsymbol{v}\times\boldsymbol{\omega}) = \frac{2}{Re}\nabla\times\nabla\times\boldsymbol{\omega}\,. \tag{2.5}$$

2.1. The velocity field equations

Expanding Eq. (2.5) using Eq. (2.3) and applying the transformation $r = e^\xi$ yields the Navier-Stokes equations in terms of the dimensionless stream function and vorticity —