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A New Preconditioned Generalised AOR Method for the Linear Complementarity Problem Based on a Generalised Hadjidimos Preconditioner

Cuiyu Liu and Chenliang Li*

School of Mathematics and Computing Science, Guilin University of Electronic Technology, Guilin, Guangxi, China, 541004.

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Abstract. A new generalised Hadjidimos preconditioner and preconditioned generalised AOR method for the solution of the linear complementarity problem are presented. The convergence and convergence rate of the new method are analysed, and numerical experiments demonstrate that it is efficient.

AMS subject classifications: 65H10, 65Y05, 65F10

Key words: Linear complementarity problem, generalised Hadjidimos preconditioner, PGAOR, M-matrix.

1. Introduction

Many researchers have studied various preconditioners to solve the well known linear algebraic system

$$Ax = b\ ,$$

so that corresponding classical iterative methods such as Jacobi or Gauss-Seidel converge faster. Hadjidimos [10] considered the preconditioner

$$P_1(\alpha) \equiv I + S_1(\alpha) = \begin{pmatrix} 1 & & & & & \\ -\alpha_2 a_{21} & 1 & & & & \\ \vdots & & \ddots & & & \\ -\alpha_i a_{i1} & & & 1 & & \\ \vdots & & & & \ddots & \\ -\alpha_n a_{n1} & & & & & 1 \end{pmatrix}, \tag{1.1}$$

*Corresponding author. *Email addresses:* `liucy@guet.edu.cn` (C. Liu), `chenliang_li@hotmail.com` (C. Li)

where $\alpha = [0, \alpha_2, \cdots, \alpha_i, \cdots, \alpha_n] \in R^n$ involves constants $\alpha_i \geq 0$, $i = 2(1)n$ and

$$S_1(\alpha) = \begin{pmatrix} 0 & & & & & \\ -\alpha_2 a_{21} & 0 & & & & \\ \vdots & & \ddots & & & \\ -\alpha_i a_{i1} & & & 0 & & \\ \vdots & & & & \ddots & \\ -\alpha_n a_{n1} & & & & & 0 \end{pmatrix}. \tag{1.2}$$

In the case where $\alpha_i = 1$, $i = 2(1)n$, $P_1(\alpha)$ is the Milaszewicz preconditioner [17], which eliminates the elements of the first column of A below the diagonal.

It has been found that preconditioner modifications can improve the convergence rates of classical iterative methods [10]. Wang [11] presented a preconditioner $P = I + S_{\alpha\beta}$, where α, β are constants and

$$S_{\alpha\beta} = \begin{pmatrix} 0 & & & & & \\ 0 & 0 & & & & \\ \vdots & & \ddots & & & \\ 0 & & & 0 & & \\ \vdots & & & & \ddots & \\ -a_{n1}\alpha - \beta & 0 & & \cdots & & 0 \end{pmatrix}. \tag{1.3}$$

If $\beta = 0$, the Wang preconditioner becomes the Evans preconditioner [7]. In this paper, we extend the Hadjidimos and Wang preconditioner approach by constructing a **generalised Hadjidimos preconditioner** $P_1(\gamma\beta) = I + S_1(\gamma\beta)$, where

$$S_1(\gamma\beta) = \begin{pmatrix} 0 & & & & & \\ -\gamma_2 a_{21} - \beta_2 & 0 & & & & \\ \vdots & & \ddots & & & \\ -\gamma_i a_{i1} - \beta_i & & & 0 & & \\ \vdots & & & & \ddots & \\ -\gamma_n a_{n1} - \beta_n & & & & & 0 \end{pmatrix}, \tag{1.4}$$

$\gamma = [0, \gamma_2, \cdots, \gamma_i, \cdots, \gamma_n] \in R^n$, $\gamma_i \geq 0, i = 2(1)n$, and β_i, $i = 2(1)n$ are constants. Thus in (1.4), if $\gamma_i = 1, i = 2(1)n$, $\beta_i = 0, i = 2(1)n$, $P_1(\gamma\beta)$ we have the Milaszewicz preconditioner, and if $\gamma_i = 0, i = 2(1)n-1$, $\beta_i = 0, i = 2(1)n-1$, $P_1(\gamma\beta)$ the Wang preconditioner.

Given the established efficiency of preconditioners for solving linear algebraic systems, in this paper we consider the solution of the linear complementarity problem [13]:

find $x \in R^n$ such that

$$x \geq 0,\ Ax - f \geq 0,\ x^\top(Ax - f) = 0\,, \tag{1.5}$$