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Longshore Submerged Wave Breaker for a Reflecting Beach

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Abstract. This paper considers the effect of a hard-wall beach on the downstream side of submerged parallel bars in a breakwater. In previous research, it was assumed that the beach can absorb all of the transmitted wave energy, when an optimal dimension for a submerged parallel bar is obtained and the wave amplitude is reduced as more bars are installed. However, for a hard-wall beach there are waves reflected from the beach that change the long-term wave interaction. We adopt the linear shallow water equations in Riemann invariant form and use the method of characteristics, in a procedure applicable to various formations of submerged rectangular bars. The distance from the parallel bar (or bars) to the beach determines the phase differences between right running waves in the beach basin and whether they superpose destructively or constructively before hitting the beach, to define the safest and the most dangerous cases. Our numerical calculations for one bar, two bars and for periodic rectangular bars confirm the analytical formulae obtained.

AMS subject classifications: 65M25, 74J20, 35L05

Key words: Method of characteristics, submerged parallel wave bars, shallow water equations.

1. Introduction

Whenever an incoming water wave enters a region where the depth suddenly changes, it scatters into a transmitted wave and a reflected wave. This mechanism underlies the concept of wave breakers, which scatter incoming waves so their amplitudes are reduced. Breakwaters consisting of submerged solid bars are often used. In Mei *et al.* [1], the optimal dimension of the one-bar submerged breakwater was determined. In Pudjaprasetya *et al.* [2], the optimal dimension of the submerged breakwater was simulated and a generalisation to consider an n-bar submerged breakwater was discussed. Wiryanto [3] studied

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wave propagation over a submerged bar by solving an appropriate linear potential problem, and his results clearly indicated there is an optimal dimension for the submerged bar. Mattioli [4] studied resonant reflection due to a series of submerged solid bars, and in particular the effect of evanescent modes. In all of this work, it is assumed that the beach can absorb all of the transmitted wave energy.

Incident waves are also scattered by another type of breakwater, the sinusoidal submerged bar. A sinusoidal bar can be a very effective scatterer due to Bragg resonance — cf. Heathershaw [5] for experimental work, and Davies & Heathershaw [6] for theory — cf. also [1]. Indeed, a rather small sinusoidal breakwater amplitude may produce a quite large reflected wave amplitude, and hence a transmitted wave of correspondingly small amplitude. Bragg resonance occurs when the incident wavelength is nearly twice the wavelength of the sinusoidal bars. Yu and Mei [7] studied the effect of shore reflection from a vertical wall, located at some distance to the right of the sinusoidal breakwater. They found that the free-surface oscillations at the wall can vary between 1 to 3.6 times the amplitude of the incident waves, depending on the distance between the breakwater and the wall.

For practical reasons, the bars in a man-made breakwater are very simple, such as rectangular. However, the case of a sinusoidal breakwater indicates the effect of shore reflection downstream from the breakwater should be studied thoroughly before construction. In this paper, we use the linear shallow water equation in Riemann invariant form to consider a monochromatic wave incident on a piecewise constant bottom topography with a reflecting (hard-wall) beach, and study the interaction between the reflected and transmitted waves. The distance between the breakwater and the beach is found to define the phase difference between the right running waves in the beach basin, and hence whether these waves superpose destructively or constructively. For a breakwater consisting of parallel rectangular bars with an optimal dimension, a formula for the safest and the most dangerous distances can be obtained. We solve the shallow water equation for the piecewise constant bottom topography numerically, using the method of characteristics. In this way, we are able to impose an incident right running monochromatic wave and simultaneously absorb the left running reflected wave, and eventually observe the long-term wave interaction. For a one-bar breakwater of a certain height, the wave amplitude in the beach basin is found to vary between 1.26 to 3.12 times the incident wave amplitude depending on the distance between the breakwater and the wall, similar to the result found for the sinusoidal submerged bar [7]. The distance between the reflecting wall and the submerged bar is therefore key to the qualitative change of the wave response. Although linear, this work provides important insight into the effect of a reflecting boundary, and may also be applicable in acoustics and optics. The analogy between water waves and optics is discussed in Andonowati & Van Groesen [8], and in Gisolf & Verschuur [9] for acoustic waves.

For clarity, let us here briefly reconsider the optimal dimension of a submerged bar. When an incident monochromatic wave with amplitude A passes a submerged bar of a certain height and width, due to reflections at the depth change the wave scatters into a transmitted wave with amplitude A_T and a reflected wave with amplitude A_R. The value