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A Hybrid FETD-FDTD Method with Nonconforming Meshes

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To the memory of David Gottlieb

Abstract. A quasi non-overlapping hybrid scheme that combines the finite-difference time-domain (FDTD) method and the finite-element time-domain (FETD) method with nonconforming meshes is developed for time-domain solutions of Maxwell's equations. The FETD method uses mixed-order basis functions for electric and magnetic fields, while the FDTD method uses the traditional Yee's grid; the two methods are joined by a buffer zone with the FETD method and the discontinuous Galerkin method is used for the domain decomposition in the FETD subdomains. The main features of this technique is that it allows non-conforming meshes and an arbitrary numbers of FETD and FDTD subdomains. The hybrid method is completely stable for the time steps up to the stability limit for the FDTD method and FETD method. Numerical results demonstrate the validity of this technique.

AMS subject classifications: 65L60, 65L12, 20B40, 83L50

Key words: Finite-element time-domain, finite-difference time-domain, discontinuous Galerkin, domain decomposition, non-conforming mesh.

1 Introduction

The finite-difference time-domain (FDTD) method [1] is an efficient and robust time-domain technique based on a structured grid and is ideal for modeling large homogeneous volumes and regular structures, while the finite-element time-domain (FETD) method [2] is more flexible in geometric modeling because of its unstructured mesh. However, the FETD method is not as popular as the FDTD method because a sparse

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matrix solution is required for time stepping, which can become very expensive even for moderate-size problems. Therefore, a hybrid FDTD-FETD method can potentially take advantages of both the flexibility of the FETD method and the efficiency of the FDTD method.

Significant efforts have been reported for hybridization of the FETD method with the FDTD method [3–9, 17, 18]. For example, the hybrid FD/FE methods has been developed to solve Maxwell's equations in the time domain, but a weak instability has been reported [3, 4]. A stable hybrid FDTD/FETD method has been presented by [5]. The FETD method utilized in these hybrid methods is based on the second-order wave equation derived from the original first-order Maxwell's equations with a single variable **E** or **H** [10]. A pyramid element is used to joint the structured FDTD grid with the unstructured FETD mesh [11]. To our knowledge, the mesh between subdomains is required to be conformal in the previous approaches.

To improve efficiency and flexibility, a non-conforming mesh between different subdomains is desirable for simulating multi-scale structures. Furthermore, multiple FETD and FDTD regions are necessary when the number of unknowns becomes large. It is the objective of this work to develop such a hybrid scheme that can allow nonconforming meshes and multiple subdomains similar to the discontinuous Galerkin spectral element time-domain (DG-SETD) method in [12]. To facilitate a non-conforming mesh, the discontinuous Galerkin method [13] would be an appropriate framework for hybridizing the FETD and FDTD methods, as the discontinuous Galerkin method allows discontinuous basis functions across different subdomains. Highly parallel discontinuous Galerkin time-domain methods have been recently employed for the solution of Maxwell's equations [13–20], but apparently this idea has not been used in the hybridization of the FETD method with FDTD method.

A new hybrid FDTD/FETD method is presented in this study. The FDTD grid and the FETD mesh can be quasi non-overlapping, i.e., they can have completely different meshes except that a buffer zone is needed to stitch them together. Furthermore, this scheme allows multiple FETD subdomains, an important feature for large-scale problems. In Section 2, we present the spatial discretization of the DG-FETD for this hybrid method. Section 3 demonstrates the coupling scheme between the FETD and FDTD methods. Numerical examples are given in Section 4 to show the validity of this method and its applications.

2 Formulation

We aim to solve for the electric and magnetic fields from Faraday's and Ampere's laws in Maxwell's equations. For the sake of simplicity without losing generality, only lossless media are considered here, although the implementation includes arbitrary conductivity losses:

$$\epsilon\frac{\partial \mathbf{E}}{\partial t}-\nabla\times\mathbf{H}=-\mathbf{J}, \tag{2.1a}$$