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REVIEW ARTICLE

Recent Developments in Numerical Techniques for Transport-Based Medical Imaging Methods

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Abstract. The objective of this paper is to review recent developments in numerical reconstruction methods for inverse transport problems in imaging applications, mainly optical tomography, fluorescence tomography and bioluminescence tomography. In those inverse problems, one aims at reconstructing physical parameters, such as the absorption coefficient, the scattering coefficient and the fluorescence light source, inside heterogeneous media, from partial knowledge of transport solutions on the boundaries of the media. The physical parameters recovered can be used for diagnostic purpose.
Numerical reconstruction techniques for those inverse transport problems can be roughly classified into two categories: linear reconstruction methods and nonlinear reconstruction methods. In the first type of methods, the inverse problems are linearized around some known background to obtain linear inverse problems. Classical regularization techniques are then applied to solve those inverse problems. The second type of methods are either based on regularized nonlinear least-square techniques or based on gradient-driven iterative methods for nonlinear operator equations. In either case, the unknown parameters are iteratively updated until the solutions of the transport equations with the those parameters match the measurements to a certain extent.
We review linear and nonlinear reconstruction methods for inverse transport problems in medical imaging with stationary, frequency-domain and time-dependent data. The materials presented include both existing and new results. Meanwhile, we attempt to present similar algorithms for different problems in the same framework to make it more straightforward to generalize those algorithms to other inverse (transport) problems.

AMS subject classifications: 35Q20, 35R30, 65N21, 65Z05, 78A46

Key words: Inverse problem, radiative transport equation, optical tomography, bioluminescence tomography, fluorescence tomography, iterative reconstruction.

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Contents

1 Introduction

Inverse problems related to the radiative transport equation have been extensively studied in recent years; see for example the reviews [14, 88–90]. In those inverse problems, the objective is to reconstruct some of the physical parameters in the transport equation inside spatial domains from partial measurements of the transport solutions on the boundaries of the domains. Apart from their applications in medical imaging, which we will address below, inverse transport problems emerge from many other areas of scientific research, including intensity modulated radiation therapy (IMRT) [8, 24], imaging in random media [15, 18], semiconductor design [31], remote sensing [6, 17, 19, 50, 103, 113–115, 119, 125, 126, 132], reactor physics [3, 82, 83] as well as ocean and atmospheric optics [29, 46, 91, 118]. For readers interested in more applications, we refer to the references [20–22, 26–28, 102, 103, 127] and the references cited there.

In this paper, we are interested in the application of inverse transport problems in medical imaging. Three major applications that we will focus on are the fields of diffuse optical tomography (DOT), fluorescence tomography (FT) and bioluminescence tomography (BLT).

Diffuse optical tomography is a biomedical imaging modality that utilizes diffuse light as a probe of tissue structure and function [9]. In diffuse optical tomography,near infra-red light are sent into biological tissues. The outgoing photon current at the surfaces of the tissues are then measured. We then want to infer the optical properties of the tissues from the knowledge of those measurements. These optical properties can be used for diagnostic purposes. Applications of optical tomography include, but not limited to, brain [40], breast [85] and joint imaging [97, 104]. We refer interested reader to [9, 11, 55, 65] for recent developments on theoretical and experimental aspects of diffusion optical tomography.

In optical molecular imaging such as fluorescence and bioluminescence tomography [81], we seek to determine the spatial concentration distribution of biological light sources inside tissues from measurements of the light current on the surface of the tissue. The light sources can come from either the fluorescent biochemical markers that we injected into the biological object, or bioluminescent cells of the object. In the former case, the markers have to be excited by an external light source, while in the later case, the cells