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Designing Several Types of Oscillation-Less and High-Resolution Hybrid Schemes on Block-Structured Grids

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Abstract. An idea of designing oscillation-less and high-resolution hybrid schemes is proposed and several types of hybrid schemes based on this idea are presented on block-structured grids. The general framework, for designing various types of hybrid schemes, is established using a Multi-dimensional Optimal Order Detection (MOOD) method proposed by Clain, Diot and Loubère [1]. The methodology utilizes low dissipation or dispersion but less robust schemes to update the solution and then implements robust and high resolution schemes to deal with problematic situations. A wide range of computational methods including central scheme, MUSCL scheme, linear upwind scheme and Weighted Essentially Non Oscillatory (WENO) scheme have been applied in the current hybrid schemes framework. Detailed numerical studies on classical test cases for the Euler system are performed, addressing the issues of the resolution and non-oscillatory property around the discontinuities.

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Key words: Hybrid schemes, oscillation-less, high resolution, MOOD, finite volume.

1 Introduction

When solving nonlinear hyperbolic conservation laws, how to achieve both the high resolution and the non-oscillatory property around the discontinuities may remain one of the most important questions. It is well known that interpolations across discontinuities tend to generate spurious oscillations that can ultimately lead to a failure of the computation. And there has been an abundance of work to deal with the conflict between keeping the high accuracy of the solutions and stabilizing the computation. Among all

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popular techniques we can cite MUSCL (monotone upstream scheme for conservation law) methods [2], weighted essentially non-oscillatory (WENO) schemes [3, 4] and central schemes [5].

MUSCL method was developed by van Leer [2] to construct high resolution total variation diminishing (TVD) schemes for solving hyperbolic conservation laws. Due to the ability to preserve stability, monotonicity as well as greater order of accuracy, MUSCL methods have become a standard widely used in todays commercial codes. For flows with strong shock waves, the use of limiters becomes necessary for preventing solution overshoots that may compromise accuracy and stability. However, it is well-known that no classical limiter has been found to work well for all problems. Indeed, some efficiently capture discontinuities but bring about some squaring effect on smoother waves, whereas other ones are accurate on smooth waves but more dissipative for sharp gradients [6]. WENO schemes utilize an adaptive stencil based on the local smoothness of the numerical solution to achieve high accuracy while avoiding oscillations near discontinuities. Higher-order WENO schemes have been constructed [7] for reducing numerical diffusion and their superior accuracy over low-order schemes for smooth flows has been demonstrated. However there are also some drawbacks of high-order WENO schemes, such as suboptimal convergence for a class of smooth solutions as well as excessive dissipation across discontinuities. For example, a characteristic-based MUSCL scheme shows better resolution for the contact discontinuity than a fifth-order WENO scheme [8]. As an alternate strategy, central schemes [5] achieve nonlinear stability through the use of artificial-dissipation models. The major difficulty in central schemes is making sure that a sufficient amount of stabilizing diffusion is added wherever it is needed to ensure stability, while in the rest of the computational domain the diffusion must be small enough not to affect the high accuracy of the scheme there. However the central schemes usually contain the artificial coefficients that are both mesh- and problem-dependent. Worse yet, there is no any guide line regarding the choice of these artificial coefficients.

A natural idea is then to develop schemes that combine the advantages of different schemes described above and avoid the disadvantages inherent in each method. Such so-called hybrid schemes have been constructed in the literature [9–14] and most of the researches utilize schemes either with spectral-like resolution [9–11] or schemes with a high order of accuracy and high efficiency in smooth regions [12–14] and then hybridize the ENO/WENO schemes to handle discontinuities. More importantly, an important component of all present hybrid schemes is using an indicator to automatically identify the discontinuity of the solution where the shock-capturing schemes can be applied. The Ref. [14] has systematically investigated a wide range of such indicators, although efficient and reliable indicators are still warranted today. Inspired by the previous work, the objective of this paper is to establish a general framework for designing various types of hybrid schemes with emphasis on achieving the high resolution and oscillation-free property around discontinuities. The novelty is to construct hybrid schemes based on a new concept of MOOD approach, which was originally proposed in [1] and further extended in the following studies [15–18]. Based on the MOOD concept, the resulting approach