Commun. Comput. Phys.
doi: 10.4208/cicp.scpde14.17s

Vol. **19**, No. 5, pp. 1317-1342
May 2016

Near-field Imaging Point-like Scatterers and Extended Elastic Solid in a Fluid

Tao Yin[1,*], Guanghui Hu[2] and Liwei Xu[1,3]

[1] *College of Mathematics and Statistics, Chongqing University, Chongqing 400044, P.R. China.*
[2] *Beijing Computational Science Research Center, Beijing 100094, P.R. China.*
[3] *Institute of Computing and Data Science, Chongqing University, Chongqing 400044, P.R. China.*

Received 9 February 2015; Accepted (in revised version) 9 September 2015

Abstract. Consider the time-harmonic acoustic scattering from an extended elastic body surrounded by a finite number of point-like obstacles in a fluid. We assume point source waves are emitted from arrayed transducers and the signals of scattered near-field data are recorded by receivers not far away from the scatterers (compared to the incident wavelength). The forward scattering can be modeled as an interaction problem between acoustic and elastic waves together with a multiple scattering problem between the extend solid and point scatterers. We prove a necessary and sufficient condition that can be used simultaneously to recover the shape of the extended elastic solid and to locate the positions of point scatterers. The essential ingredient in our analysis is the outgoing-to-incoming (OtI) operator applied to the resulting near-field response matrix (or operator). In the first part, we justify the MUSIC algorithm for locating point scatterers from near-field measurements. In the second part, we apply the factorization method, the continuous analogue of MUSIC, to the two-scale scattering problem for determining both extended and point scatterers. Numerical examples in 2D are demonstrated to show the validity and accuracy of our inversion algorithms.

AMS subject classifications: 74J25, 74J20, 35R30, 35Q74

Key words: MUSCI, near-field imaging, point sources, factorization method, inverse scattering, fluid-solid interaction problem.

1 Introduction

The time-reversal imaging with Multiple Signal Classification (time-reversal MUSIC) is well-known for signal-processing applications [12,16]. It provides a method to determine

*Corresponding author. *Email addresses:* `taoyin_cqu@163.com` (T. Yin), `hu@csrc.ac.cn` (G. Hu), `xul@cqu.edu.cn` (L. Xu)

one or more unknown small scatterers from the so-called multistatic response matrix by neglecting the physical properties and the geometry of the target. Several algorithms have been developed to recover extended scatterers since then. The approach proposed in [17, 38] was based on a physical factorization of the scattered field and the singular value decomposition of the response matrix for extended targets. This approach has been extended to recover point and extended scatterers from far-field patterns [5], relying on a generalized Foldy-Lax formulation proposed originally in [15, 23].

The Factorization Method [28] proposed by Kirsch (1998) can also be regarded as the continuous analogue of the MUSIC algorithm. A relation between the MUSIC and Linear Sampling Method was first investigated in [7]. In the Born approximation case where the multiple scattering between the point scatterers are neglected, the MUSIC was treated as a discrete analogue of the Factorization Method in inverse medium scattering [27]. Consequently, the range of the far-field response matrix can be used to derive a necessary and sufficient condition for precisely characterizing the positions of point scatterers. The same characterization was obtained in [8,9] by taking into account the multiple scattering in the Foldy regime. The direct and inverse electromagnetic scattering by isotropic point-like obstacles in three dimensions were analyzed in [10], and the determination of the scattering strengths attached to point scatterers has been discussed in [8–10].

The goal of this paper is to justify the MUSIC algorithm for recovering point-like and extended scatterers from the near-field measurements generated by incident point sources. Compared to the far-field case (see e.g., [8, 9, 27] or [29, Chapter 4.1]), difficulty arises from the failure of the decomposition of the near field matrix (or operator) $\mathbf{N}$ into the from $\mathbf{N}=\mathbf{H}^*\mathbf{S}\mathbf{H}$ (cf. Sections 2.2 and 3.1). On the other hand, it is an open problem how to factorize the near-field operator analogously to the far-field case until the recent studies of the outgoing-to-incoming (OtI) operator carried out in [21]. Following the idea of [21] we apply the OtI operator T to the near-field response matrix (or operator) and hence obtain a factorization of the form $T\mathbf{N}=\mathbf{H}^*\mathbf{S}\mathbf{H}$. Consequently, the spectrum of the modified near-field response matrix (or operator) $T\mathbf{N}$ can be used to characterize the scatterers, even in the two-scale scattering model. We emphasize that the uniqueness follows immediately from our computational criterion, because it is not only sufficient but also necessary for solving the inverse problem.

In this paper, an appropriate factorization of the near-field operator is established with the help of the 'impedance' boundary conditions across point-like scatterers (see (2.4)-(2.5)). These boundary conditions show that, around a point scatterer, the behavior of total field is similar to that of a point source wave, and that the coefficient of the leading (singular) term is proportional to that of the sub-leading term (that is why they look like the impedance boundary condition for extended obstacles). Our methods are closest to the recently developed imaging scheme [22] for inverse acoustic scattering by an extended sound-soft obstacle surrounded by point-like scatterers. Emphasis of this paper will be placed upon a straightforward proof of the well-posedness of direct scattering problems, providing 'explicit' solutions to the fluid-solid interaction problem in the two-scale model. Our mathematical analysis turns out to be more complicated and tricky