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Solving Maxwell's Equation in Meta-Materials by a CG-DG Method

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Abstract. In this paper, an approach combining the DG method in space with CG method in time (CG-DG method) is developed to solve time-dependent Maxwell's equations when meta-materials are involved. Both the unconditional L^2-stability and error estimate of order $\mathcal{O}\left(\tau^{r+1}+h^{k+\frac{1}{2}}\right)$ are obtained when polynomials of degree at most r is used for the temporal discretization and at most k for the spatial discretization. Numerical results in 3D are given to validate the theoretical results.

AMS subject classifications: 35Q61, 65M12, 65M60

Key words: Maxwell's equations, meta-materials, full-discretization, CG-DG method, L^2-stability, L^2-error estimate.

1 Introduction

Meta-material is often referred as an artificially constructed electromagnetic material whose refraction index is negative. It was first theoretically proposed by Veselagi in 1967 [29], and practically constructed by Smith in 2000 [27]. Since then, the study of meta-materials has been a very hot topic due to its potential application in many areas, such as the design of invisibility and sub-wavelength imaging.

The numerical simulation of meta-materials plays an important role in the design of new meta-materials and the discovery of new phenomenon of meta-materials [8]. Among them, the widely used numerical methods for the simulation of meta-materials are the finite difference time domain methods (FDTD) [13, 28], the finite element methods (FEM) [21] and the commercial packages, such as HFSS and COMSOL, et al.. However, it is well known that the FDTD method has difficulties in solving problems with

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complex geometry. In the literature, there exist some works on finite element time domain (FETD) method for Maxwell's equation in dispersive media [2] and meta-materials [21, 22]. Their main advantage lies in their ability to handle arbitrary geometries via unstructured meshes of the domain of interest.

Since the development of the discontinuous Galerkin (DG) method in 1973 [26], it has become one of the most important methods in solving various kinds of differential equations. It can keep almost all the advantages of finite element method. Further, the DG method has many nice properties, e.g., applicability for non-conforming mesh, high-order accuracy, flexibility in handling material interface and high parallelizability. We refer to the survey paper [1, 7], the monographs [5, 9, 15] and their references therein for more details about it.

In recent years, DG methods have been used to solve Maxwell's equation in free space [4, 6, 10, 12, 14] and dispersive media [16, 17, 24, 30, 31], whose permittivity depends on the wave frequency. In [30] and [33], Xie et al. propose a semi-discrete DG method for dispersive media and space-time DG method for Maxwell's equation in free space. They prove that the convergence order is $\mathcal{O}(h^{k+\frac{1}{2}})$ for the semi-discrete DG method and $\mathcal{O}(\tau^{r+1}+h^{k+\frac{1}{2}})$ for the space-time DG method. It is worthwhile to point out that in the numerical examples of [30], the continuous Galerkin finite element method is used in the temporal discretization. Nevertheless, the convergence rate for the temporal variable is missing. To our knowledge this is the first effort to solve Maxwell's equation by the finite element methods (CG or DG) both for temporal and spatial discretization. Very recently, they extend this space-time DG method to dispersive media and provide the corresponding theoretical analysis and numerical results [31].

Recently, Li in [18] develops an approach using DG approach for spatial discretization and Runge-Kutta (finite difference) method in temporal discretization to solve the time-domain Maxwell's equation in meta-materials. Although several numerical examples are given to show that this method is efficient, the theoretical analysis is missing. In [23], Li et al. develop a leap-frog type DG method for solving the time-domain Maxwell's equation in meta-materials, and prove the stability and convergence of it. In [20], Li et al. develop a leap-frog type DG method for solving the time-domain Maxwell's equation in meta-materials based on auxiliary differential equation (ADE) method, and prove that, under some CFL condition, this method is stable and convergent. In [32], Wang et al. develop an implicit DG method for solving time-domain Maxwell's equations in meta-materials based on a system of integro-differential equations. They prove that the implicit DG scheme is unconditionally stable and the convergence rate is $\mathcal{O}(\tau^2+h^{k+\frac{1}{2}})$. However, the implicit DG scheme cannot make full use of the advantages of high order DG method.

Motivated by the works in [20, 30, 33], we introduce a method combining the DG method in space with the CG method in time (CG-DG method) for Maxwell's equation in meta-materials. Our work is based on a so-called auxiliary differential equation (ADE) method. The key point of it is to introduce another set of differential equations to include media interactions. We prove that this CG-DG scheme is unconditionally stable and con-