Commun. Comput. Phys.
doi: 10.4208/cicp.091014.060315a

Vol. **18**, No. 5, pp. 1313-1335
November 2015

An Adaptive Combined Preconditioner with Applications in Radiation Diffusion Equations

Xiaoqiang Yue[1], Shi Shu[1,*], Xiaowen Xu[2] and Zhiyang Zhou[1]

[1] *School of Mathematics and Computational Science, Xiangtan University, Hunan 411105, P.R. China.*
[2] *Institute of Applied Physics and Computational Mathematics, Beijing 100088, P.R. China.*

Received 9 October 2014; Accepted (in revised version) 6 March 2015

Abstract. The paper aims to develop an effective preconditioner and conduct the convergence analysis of the corresponding preconditioned GMRES for the solution of discrete problems originating from multi-group radiation diffusion equations. We firstly investigate the performances of the most widely used preconditioners (ILU(k) and AMG) and their combinations (B_{co} and $\tilde{B}_{co}$), and provide drawbacks on their feasibilities. Secondly, we reveal the underlying complementarity of ILU(k) and AMG by analyzing the features suitable for AMG using more detailed measurements on multiscale nature of matrices and the effect of ILU(k) on multiscale nature. Moreover, we present an adaptive combined preconditioner B_{co}^{α} involving an improved ILU(0) along with its convergence constraints. Numerical results demonstrate that B_{co}^{α}-GMRES holds the best robustness and efficiency. At last, we analyze the convergence of GMRES with combined preconditioning which not only provides a persuasive support for our proposed algorithms, but also updates the existing estimation theory on condition numbers of combined preconditioned systems.

AMS subject classifications: 65F10, 65F15, 65N55, 65Z05

Key words: Radiation diffusion equations, combined preconditioner, adaptive preconditioning, convergence analysis.

1 Introduction

Equations of radiation hydrodynamics (RHE), which play a quite important role in numerical simulations of inertial confinement fusion (ICF), contain a set of coupled equations that describe the motion of fluids and radiative transition processes, where the former is described by a multi-material Eulerian hydrodynamic equations while the latter

*Corresponding author. *Email addresses:* yuexq@xtu.edu.cn (X. Yue), shushi@xtu.edu.cn (S. Shu), xwxu@iapcm.ac.cn (X. Xu), peghoty@163.com (Z. Zhou)

approximatively by multi-group radiation diffusion (MGD) equations [1]. It is a widely-used way to discretize the hydrodynamic equations explicitly and implicitly for MGD equations, which gives rise to large-scale sparse linear systems solved for high resolutions. It should be stressed that the computational costs occupy the predominantly time-consuming part (more than 80% in general), numerical solutions of discretized algebraic systems become the crucial bottleneck in ICF simulations. The preconditioned iterative method is a primary choice to handle them, although its efficiency is affected severely by the preconditioner.

In recent years, numerous preconditioners constructed for radiation diffusion equations have been presented, see [2–10] and references therein for details. These preconditioners mainly fall into several preconditioning algorithms, such as incomplete LU factorization (ILU) [11,12] and algebraic multigrid method (AMG) [13,14], which will be called standalone preconditioners below. However, the actual physical modelings of ICF are becoming more and more meticulous along with the development of high-performance computers and the capability of numerical simulations. The existing standalone preconditioners are difficult or even no more to adapt to numerical simulations of complex physical modelings, which is mainly due to the following two aspects: (i) they are not designed for MGD but for the 3-temperature heat-conduction equations, hard to handle the complex coupling relationships in MGD; (ii) they usually lack inconsistency with respect to the different matrices from nonlinear iterations at each integration time-step, nearly impossible to accurately simulate a problem with up to 10^5 integration time-steps.

More recently, Hu et al. [15] proposed a combined preconditioner B_{co}, which was successfully employed to solve symmetric and positive definite (SPD) linear systems arising from petroleum reservoir simulations. On this basis, we develop an adaptive combined preconditioner B_{co}^{α}, and apply it to MGD equations and further an actual numerical simulation of ICF implosion. We firstly introduce the popular strength measurement ψ and define the distribution measurements ϕ and ϱ on the multiscale nature of an arbitrary matrix, where ϱ is the number of magnitude intervals, and ϕ reflects how close these magnitude intervals are. Secondly, we obtain three adaptability conditions ensuring the feasibility of AMG by an investigation of two-dimensional convection-diffusion equations and classical one-group and twenty-group problems. Meanwhile we provide the fact that ILU(k) can weaken the multiscale strength and concentrate corresponding distributions, which brings to light their underlying complementarity and offers theoretical help to the effectiveness of combined preconditioners. Moreover, we present an improved ILU(k) by adopting a simpler criterion to filtrate tiny matrix elements prior to performing ILU(k) factorization, namely I$\hat{L}\hat{U}$(k) in this study, which is a subtle version roughly equivalent to ILU(k) but with a notable reduction in memory storage and computational cost when disposing of multiscale problems. B_{co}^{α} is subsequently achieved by an additional heuristic remedy on I$\hat{L}\hat{U}$(0)-GMRES. Our numerical results verify the considerable advantage of B_{co}^{α} over others.

In addition, we obtain a qualitative convergence estimation of a nonsymmetric combined preconditioner. Its analysis procedure consists of a transformation into the estima-